Rise Time

The response of an RC circuit to a voltage step of amplitude V_0, starting at time t=0 can be characterized by the time constant $\tau = R.C$ [s].

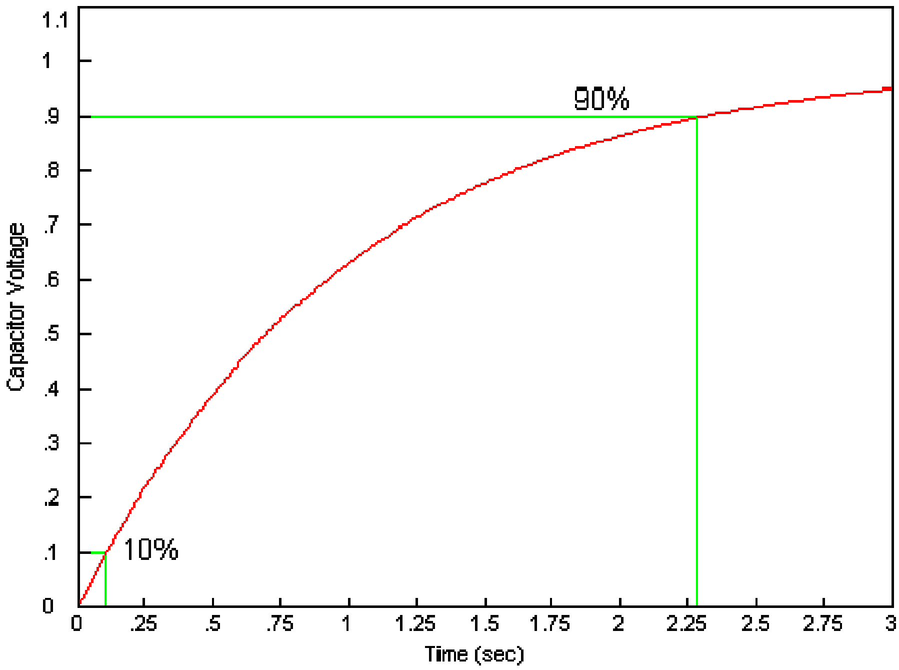
$$v(t) = V_0 \left(1 - e^{-t/\tau} \right) \quad \text{for } t \geq 0$$

At the time $t = \tau$ the voltage rises to a value given by $V_0\ (1-e^{-1})$. This is about 63% of the full value V_0. It takes approximately 4 more time constants to reach the full value to a good degree of accuracy (less than 1% error).

A related measurement is that of 'rise time'.

The definition of rise time T_{RT} is "that time taken for a linear network's output to rise from 10% to 90% of its final value when stimulated by a step input". This measurement is useful because it is easy to measure on an oscilloscope and can be applied to any linear network.

For our example of τ = 1 second and for a 1 volt input step we have the following:



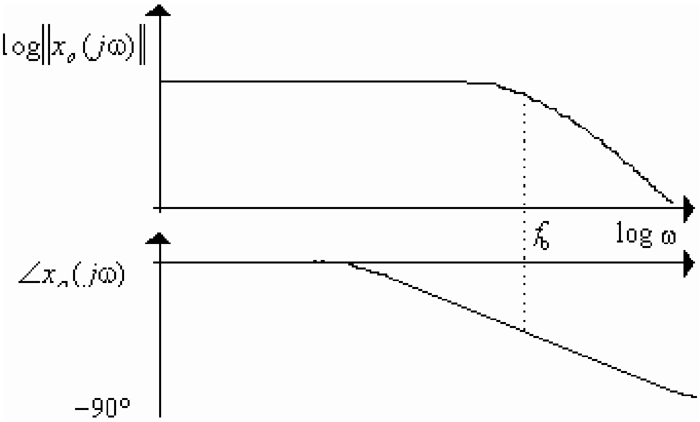
In case of the RC network, the 10% level is reached after 0.105 time constants and the 90% after 2.302 time constants; thus the rise time is (2.302 – 0.105) time constants which is 2.197 time constants or 2.197τ. We'll call it 2.2τ for simplicity.

$$T_{RT} = 2.2\tau$$

In the frequency domain, the RC network offers no attenuation or loss at DC (0 Hz);

The attenuation rises with frequency according to the formula

$$x_0 \equiv V(\omega)/V(0) = (1 + (\omega/\tau)^2)^{-2}$$



Using the dB logarithmic scale where the <u>attenuation</u> X_0 is expressed in dB as

$$-20 \log [V(\omega)/V(0)]$$

The 3 dB attenuation is reached at the (corner) frequency of $\omega_1 \equiv 1/(2\pi RC)$ or $1/(2\pi\tau)$.

This frequency is usually referred to as the <u>3dB bandwidth</u> or the <u>half power bandwidth</u> because at this frequency $V^2(\omega_1)/V^2(0) = 0.5$. From this point on the output voltage keeps dropping at a rate of <u>6dB per octave</u>.(An octave means a frequency increase that doubles the current frequency value).

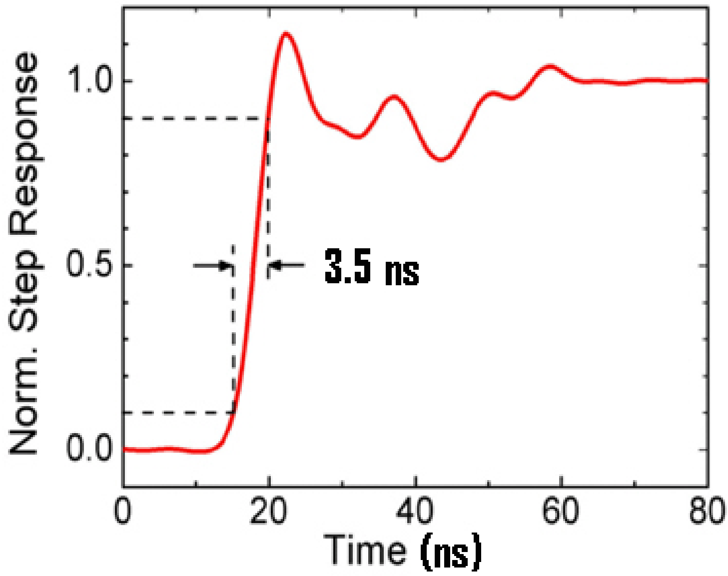
Now multiplying the 3 dB frequency by the rise time:

$$1/(2\pi\tau) \times 2.2\tau = 2.2/2\pi = 0.35 \text{ (dimensionless)}$$

OR:

Bandwidth x Rise Time = 0.35

The figure of 0.35 often is quoted when characterizing the performance of oscilloscopes.



A 'scope whose bandwidth is 100 MHz and whose step response is that of a simple RC network will have an internal rise time of 3.5 ns. Therefore, a pulse with rise time shorter than this cannot be measured on such a 'scope and it is fruitless to attempt to measure the rise time of a pulse if it is shorter than that of the 'scope's. Even signals whose rise time approaches that of the 'scope will be distorted so that their measured rise time is increased. The measured rise time is given by:

$$rt_{measured} = \{(rt_{scope})^2 + (rt_{pulse})^2\}^{½}$$

Rule: if you want to measure the rise time of a pulse; know what the rise time of your 'scope is otherwise you may end up merely measuring that of the 'scope and not that of the pulse.

<u>Problem</u>

An electronic device receives data at an average bit rate of 10MB/s. The amplitude of data pulses is 3.3 V. The data sheet specifies that in order to guarantee the data validity the minimum voltage slope of the data pulses has to be 100 ns/V. The minimum pulse amplitude specified is 1.6V.

a. What should be the minimum bandwidth of the data channel?
b. What measures are open to the designer to reduce the data channel bandwidth and by how much?

Solution:

Step1: Calculate the minimum Rise Time. From 0V to 3.3V level it takes > 3.3x100 ns =330 nS. T_{RT} = 0.8x330 nS = 2.64 nS.

Step 2: Calculate the Bandwidth: BW $= 0.35/T_{RT}$

$= 0.35/2.64$ nS $= 0.132 \times 10^9$ Hz = 132 MHz!

The BW can be reduced by reducing the pulse amplitude from 3.3V to 1.6V. This will halve the required bandwidth.