Printable collection of Bode Plot Web Pages

This document is a compilation of all of the Bode plot pages in one document for convenient printir

Contents

Printable collection of Bode Plot Web Pages

This document is a compilation of all of the Bode plot pages in one document for convenient printing.

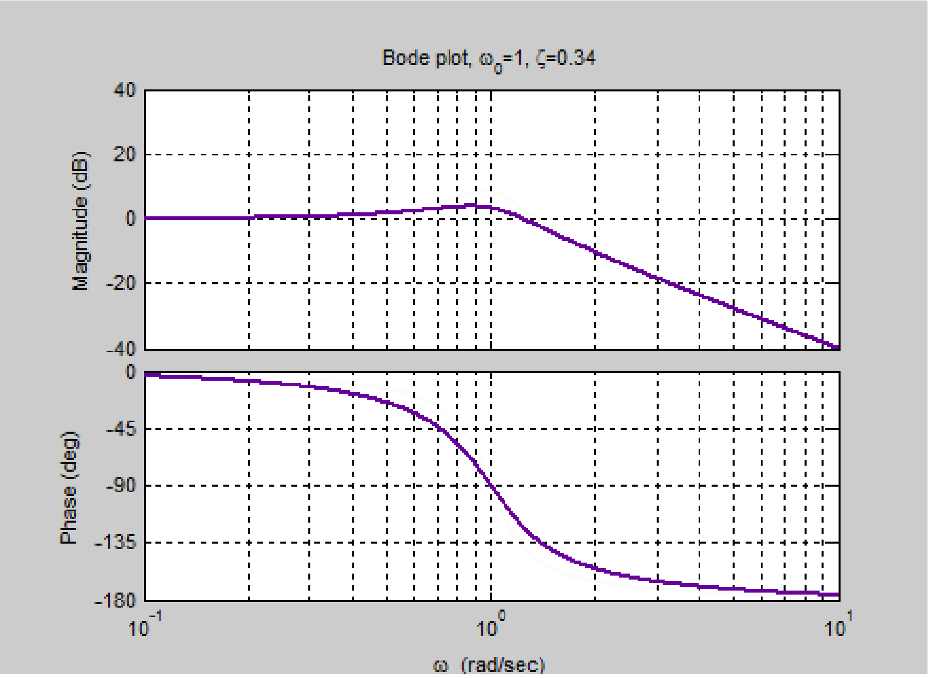
Contents

- Introduction
- The Frequency Domain: What do Bode plots represent?
- The Asymptotic Plot: Defining the rules for making sketches.
- The Method: Applying the rules to make sketches.
- Examples: A series of Examples.
- Rules Redux: A compact representation of the rules (including a pdf).
- BodePlotGui: A MatLab GUI that helps to explain the method.
- BodePaper: A MatLab function that will create the plots necessary for making sketche hand.

Bode Plots Overview

Overview | Freq Domain | Asymptotic plots | Making Plots | Examples | Drawing Tool | BodePlotGui | Rules Table | Printable

<< Stop >>



Bode plots are a very useful way to represent the gain and phase of a system as a function of frequency. This is referred to as the frequency domain behavior of a system. This web page attempts to demystify the process. The various parts are more-or-less stand alone, so if you want to skip one or more, that should not be a problem. If you are only interested in a quick lesson on how to make Bode diagrams, go to "Making Plots." A a quick lesson on how to make Bode diagrams, go to "Making Plots." A MATLAB program to make piecewise linear Bode plots is described in BodePlotGui.

The documents are:

1. What is the frequency domain response? In other words, "What does a Bode Plot represent?" This includes an animation.
2. How are the piecewise linear asymptotic approximations derived?
3. Rules for making Bode plots. This is a quick "How to" lesson for drawing Bode plots.
4. Some examples (1, 2, 3, 4, 5, 6) - (combined into one file).
5. BodePlotGui: A software tool for generating asymptotic Bode plots.
6. A MatLab program for making semi-logarithmic paper for drawing your own Bode plots.
7. A table summarizing Bode rules
8. The MATLAB files discussed in these documents

What Bode Plots Represent: The Frequency Domain

Overview | Freq Domain | Asymptotic plots | Making Plot | Examples | Drawing Tool | BodePlotGui | Rules Table | Printable

Contents

Why Sine Waves?

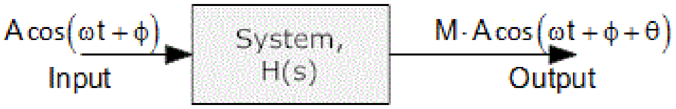
One of the most commonly used test functions for a circuit or system is the sine (or cosine) wave. This is not because sine waves are a particularly common signal. They are in fact quite rare - the transmission of electricity (a 60 Hz sine wave in the U.S., 50 Hz in much of the rest of the world) is one example. The reason sine waves are important is complex and involve a branch of Mathematics called Fourier Theory. Briefly put: any signal going into a circuit can be represented by a sum of sinusoidal waves of varying frequency and amplitude (often an infinite sum).

This is why sine waves are important. Not because they are common, but because we can represent arbitrarily complex functions using only these very simple function.

Determining system output given input and transfer function

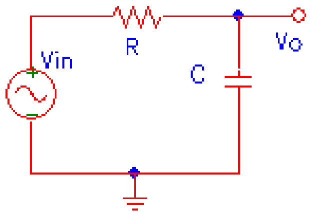
Given that sinusoidal waves are important, how can we analyze the response of a circuit or system to sinusoidal inputs (after all transients have died out - the so-called sinusoidal steady state)? There are many ways to do this, depending on your mathematical sophistication. Let's use a fairly basic explanation that uses phasors. If you are unfamiliar with phasors, a brief introduction is here. A technique using Laplace Transforms is given here.

For a system of the type we are studying (linear constant coefficient) if the input to a system is sinusoidal at a particular frequency, then the output of the system is also a sinusoid at the same frequency, but typically with a different amplitude or phase. Put another way, if the input to a system (described by the transfer function H(s)) is A·cos(ω·t+ϕ) then the output is M·A·cos(ω·t+ϕ+θ). This is likewise true for sine, since it simply a cosine with ϕ=-π2 radians (or -90°). This is shown below.



In this diagram the magnitude of the sinusoid has changed by a factor of M (which we will take to be a positive real number) and the phase has changed by a factor of θ (a real number, not necessarily positive). It is our task to find the value of M and θ for a particular system, H(s), at a particular frequency, ω. We call M the magnitude of the system (or transfer function) at ω, and we call θ the phase of the system at that frequency.

Using complex impedances it is possible to find the transfer function of a circuit. For example, the circuit below is described by the transfer function, H(s), where s= *jω*.

Circuit	Transfer Function
	$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1+sRC}$

Consider the case where R=2MΩ and C=1µF. In that case:

$$H(s) = \frac{1}{1+2s} \qquad H(j\omega) = \frac{1}{1+j2\omega}$$

Generally we know the input V_{in} and want to find the output V_{out}. We can do this by simple multiplication

$$V_{out}\left(j\omega\right) = V_{in}\left(j\omega\right) \cdot H(j\omega) = V_{in}\left(j\omega\right) \cdot \frac{1}{1+j2\omega}$$

If we have a phasor representation for the input and the transfer function, the multiplication is simple (multiply magnitudes and add phases). Finding the output becomes easy. Try it out.

Interactive Demo

Choose a transfer function.

●

$H\left(s\right) = \frac{1}{1+2s}$ $H\left(j\omega\right) = \frac{1}{1+j2\omega}$

○

$H\left(s\right) = \frac{1.6}{s^2+0.5s+1.6}$ $H\left(j\omega\right) = \frac{1.6}{(1.6-\omega^2)+j(0.5\cdot\omega)}$

Set input parameters,
$V_{in}(t)=A\cdot\cos(\omega\cdot t+\phi)$.

Set ω: 1.000 ω
0
3

Set A: 1 A
0.2
2

Set ϕ: 0 ϕ
-180
180

At ω =1, H(*j*ω) = 1/(1.00 + j2.00) = 0.45∠-63.4° = M∠θ.
Since the input can be represented as 1∠0°,
The output is
M·A∠(θ+ϕ) =
0.45∠-63.4°.

	Magnitude	Phase	Time Domain
H(*j*ω)	0.45	-63.4°	0.45·cos(1·t + -63.4°)
Input	1	0°	1·cos(1·t + 0°)
Output	0.45	-63.4°	0.45·cos(1·t + -63.4°)

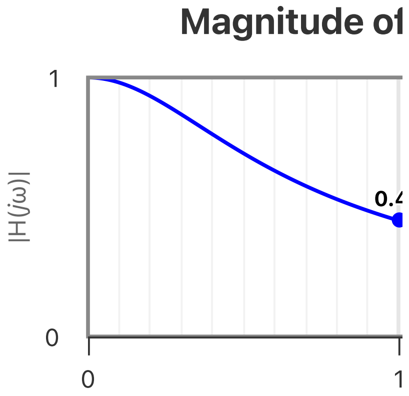
Directions for Use

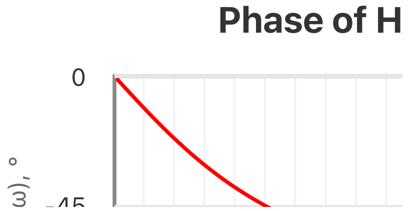
Directions for Use

Use the radio buttons to choose a transfer function, and the sliders to choose the frequency, amplitude and phase of the input (you can also set frequency by clicking and dragging in either of the top two graphs.)

The paragraph below the sliders goes through the calculation of the numerical value of the transfer function at the chosen frequency, and gives H(*j*ω) in terms of magnitude and phase. Note that these are also shown on the top two graphs by a dot. To find the magnitude of the output, simply multiply the magnitude of the input (A) by the magnitude of the transfer function (M). The phase of the output is sum of the input phase (ϕ) and the phase of the transfer function (θ).

The bottom graph shows input, $V_{in}(t)$ in black, and $V_{out}(t)$ in magenta. The period, T (maroon), is shown from one upward zero-crossing of the input function to the next (shown by black dots). The delay T_d (green), is shown from an upward zero crossing of the input to the next upward zero crossing of the output (green dot). The phase is negative (since output lags input) and equal to -T_d/T·360°. So if the delay was T_d=T/4 (i.e., one quarter of a period) the phase shift would be -90°)





The Asymptotic Bode Diagram: Derivation of Approximations

Overview | Freq Domain | Asymptotic plots | Making Plot | Examples | Drawing Tool | BodePlotGui | Rules Table | Printable

Skip ahead to interactive demos.

Contents

Skip ahead to interactive demos.

Introduction

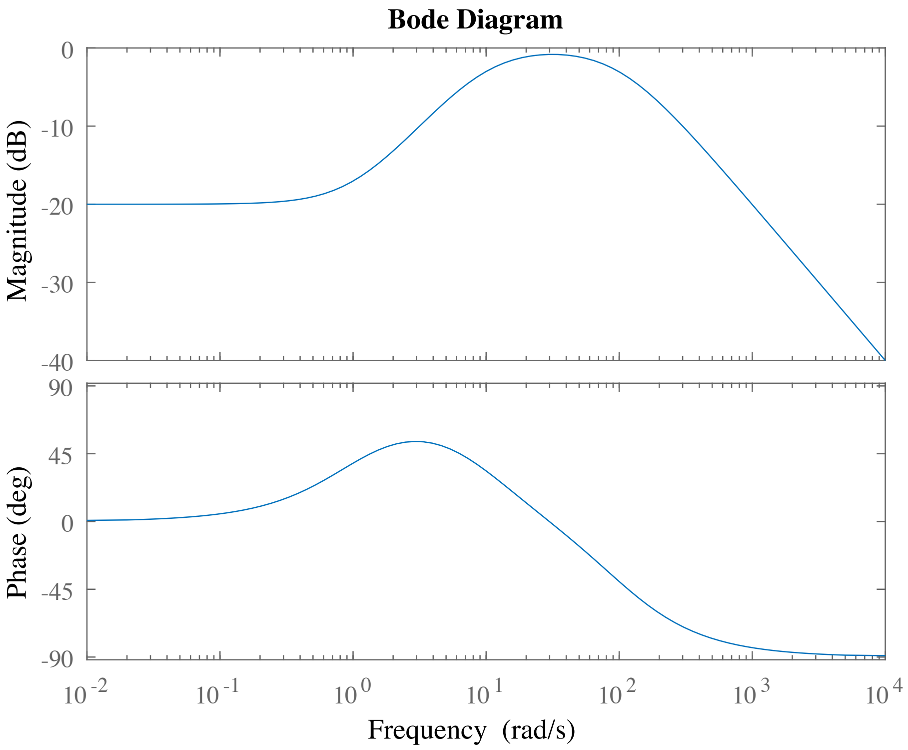
Given an arbitrary transfer function, such as

$$H(s) = \frac{100s + 100}{s^2 + 110s + 1000}$$

if you wanted to make a mode plot you could calculate the value of H(s) over a range of frequencies (recall s=j·ω for a Bode plot), and plot them. This is what a computer would naturally do. For example if you use MATLAB® and enter the commands

```
>> mySys=tf(100*[1 1],[1 110 1000])
mySys =
     100 s + 100
  ------------------
  s^2 + 110 s + 1000
>> bode(mySys)
```

you get a plot like the one shown below. The asymptotic solution is given elsewhere.



However, there are reasons to develop a method for sketching Bode diagrams manually. By drawing the plots by hand you develop an understanding about how the locations of poles and zeros effect the shape of the plots. With this knowledge you can predict how a system behaves in the frequency domain by simply examining its transfer function. On the other hand, if you know the shape of transfer function that you want, you can use your knowledge of Bode diagrams to generate the transfer function.

The first task when drawing a Bode diagram by hand is to rewrite the transfer function so that all the poles and zeros are written in the form (1+s/ω_0). The reasons for this will become apparent when deriving the rules for a real pole. A derivation will be done using the transfer function from above, but it is also possible to do a more generic derivation. Let's rewrite the transfer function from above.

$$H(s) = 100\frac{s+1}{(s+10)(s+100)} = 100\frac{1+s/1}{10\cdot(1+s/10)\cdot 100\cdot(1+s/100)}$$
$$= 0.1\frac{1+s/1}{(1+s/10)(1+s/100)}$$

Now let's examine how we can easily draw the magnitude and phase of this function when s=*jω*.

First note that this expression is made up of four terms, a constant (0.1), a zero (at s=-1), and two poles (at s=-10 and s=-100). We can rewrite the function (with s=*jω*) as four individual phasors (i.e., magnitude and phase), each phasor is within a set of square brackets to make them more easily distinguished from each other..

$$H(j\omega) = 0.1\frac{1+j\omega/1}{(1+j\omega/10)(1+j\omega/100)}$$
$$= [|0.1|\angle(0.1)]\,\frac{[|1+j\omega/1|\angle(1+j\omega/1)]}{[|1+j\omega/10|\angle(1+j\omega/10)]\,[|1+j\omega/100|\angle(1+j\omega}$$

We will show (below) that drawing the magnitude and phase of each individual phasor is fairly straightforward. The difficulty lies in trying to draw the magnitude and phase of the more complicated function, H(*jω*). To start, we will write H(*jω*) as a single phasor:

$$H(j\omega) = \left(|0.1|\,\frac{|1+j\omega/1|}{|1+j\omega/10|\,|1+j\omega/100|}\right)(\angle(0.1)+\angle(1+j\omega/1)-\angle($$
$$= |H(j\omega)|\,\angle H(j\omega)$$

$$|H(j\omega)| = |0.1|\,\frac{|1+j\omega/1|}{|1+j\omega/10|\,|1+j\omega/100|}$$
$$\angle H(j\omega) = \angle(0.1)+\angle(1+j\omega/1)-\angle(1+j\omega/10)-\angle(1+j\omega/100)$$

Drawing the phase is fairly simple. We can draw each phase term separately, and then simply add (or subtract) them. The magnitude term is not so straightforward because the magnitude terms are *multiplied*, it would be much easier if they were added - then we could draw each term on a graph and just *add* them. We can accomplish this by usinga logarthmic scale (so multiplication and division become addition and subtraction). Instead of a simple logarithm, we will use a deciBel (or dB) scale.

A Magnitude Plot

One way to transform multiplication into addition is by using the logarithm. Instead of using a simple logarithm, we will use a deciBel (named for Alexander Graham Bell). *(Note: Why the deciBel)* The relationship between a quantity, Q, and its deciBel representation, X, is given by:

$$X = 20 \cdot log_{10}(Q)$$

So if Q=100 then X=40; Q=0.01 gives X=-40; X=3 gives Q=1.41; and so on.

If we represent the magnitude of H(s) in deciBels several things happen.

$$|H(s)| = |0.1| \frac{|1 + j\omega/1|}{|1 + j\omega/10|\,|1 + j\omega/100|}$$

$$20 \cdot \log_{10}(|H(s)|) = 20 \cdot \log_{10}\left(|0.1| \frac{|1 + j\omega/1|}{|1 + j\omega/10|\,|1 + j\omega/100|}\right)$$

$$= 20 \cdot \log_{10}(|0.1|) + 20 \cdot \log_{10}(|1 + j\omega/1|) + 20 \cdot \log$$

$$= 20 \cdot \log_{10}(|0.1|) + 20 \cdot \log_{10}(|1 + j\omega/1|) - 20 \cdot \log$$

The advantages of using deciBels (and of writing poles and zeros in the form (1+s/ω_0)) are now revealed. The fact that the deciBel is a logarithmic term transforms the multiplications and divisions of the individual terms to additions and subtsractions. Another benefit is apparent in the last line that reveals just two types of terms, a constant term and terms of the form 20·log10(|1+jω/ω_0|). Plotting the constant term is trivial, however the other terms are not so straightforward. These plots will be discussed below. However, once these plots are drawn for the individual terms, they can simply be added together to get a plot for H(s).

A Phase Plot

If we look at the phase of the transfer function, we see much the same thing: The phase plot is easy to draw if we take our lead from the magnitude plot. First note that the transfer function is made up of four terms. If we want

$$\angle H(s) = \angle(0.1) + \angle(1 + j\omega/1) - \angle(1 + j\omega/10) - \angle(1 + j\omega/100)$$

Again there are just two types of terms, a constant term and terms of the form ($1+j\omega/\omega_0$). Plotting the constant term is trivial; the other terms are discussed below.

A more generic derivation

The discussion above dealt with only a single transfer function. Another derivation that is more general, but a little more complicated mathematically is here.

Making a Bode Diagram

Following the discussion above, the way to make a Bode Diagram is to split the function up into its constituent parts, plot the magnitude and phase of each part, and then add them up. The following gives a derivation of the plots for each type of constituent part. Examples, including rules for making the plots follow in the next document, which is more of a "How to" description of Bode diagrams.

A Constant Term

Consider a constant term: $H(s) = H(j\omega) = K$

Magnitude

Clearly the magnitude is constant as ω varies. $|H(j\omega)| = |K|$

Phase

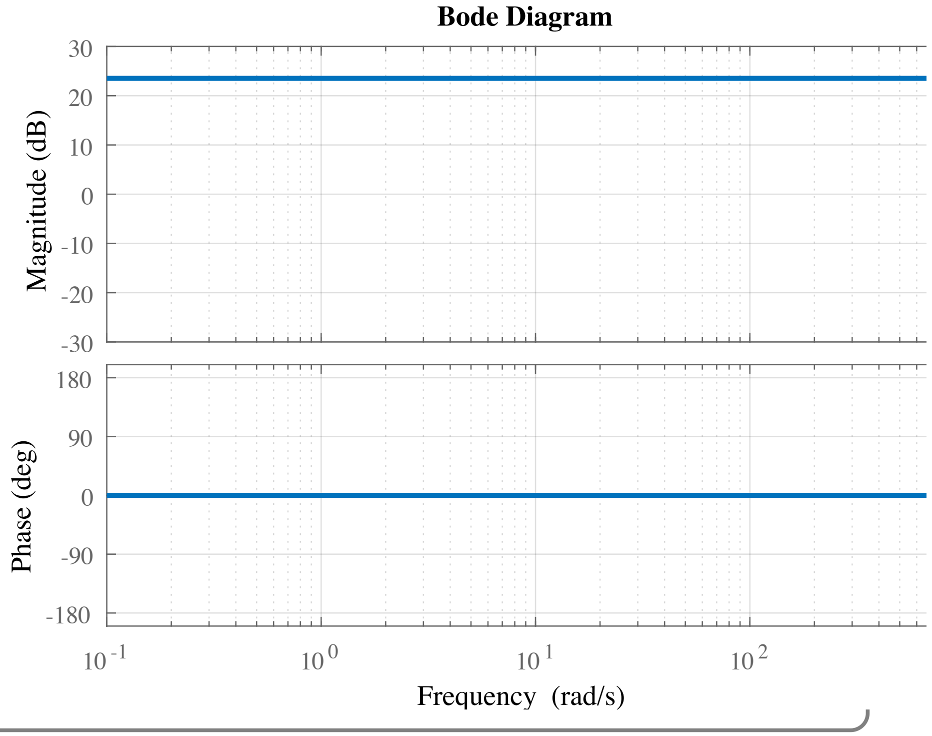
The phase is also constant. If K is positive, the phase is 0° (or any even multiple of 180°, i.e., ±360°). If K is negative the phase is -180°, or any odd multiple of 180°. We will use -180° because that is what MATLAB® uses. Expressed in radians we can say that if K is positive the phase is 0 radians, if K is negative the phase is -π radians.

Example: Bode Plot of Gain Term

$$H(s) = H(j\omega) = 15$$
$$|H(j\omega)| = |15| = 15 = 23.5\,dB$$
$$\angle H(j\omega) = \angle 15 = 0^{\circ}$$

The magnitude (in dB is calculated as

$$20 \cdot \log_{10}(15) = 23.5$$



Key Concept: Bode Plot of Gain Term

- For a constant term, the magnitude plot is a straight line.
- The phase plot is also a straight line, either at 0° (for a positive constant) or ±180° (for a negative constant).

Interactive Demo

A Real Pole

Consider a simple real pole : $H(s) = \frac{1}{1+\frac{s}{\omega_0}}, \qquad H(j\omega) = \frac{1}{1+j\frac{\omega}{\omega_0}}$

The frequency ω_0 is called the break frequency, the corner frequency or the 3 dB frequency (more on this last name later). The analysis given below assumes ω_0 is positive. For negative ω_0 here.

Magnitude

The magnitude is given by

$$|H(j\omega)| = \left|\frac{1}{1+j\frac{\omega}{\omega_0}}\right| = \frac{1}{\sqrt{1^2+\left(\frac{\omega}{\omega_0}\right)^2}}$$

$$\left|H\left(j\omega\right)\right|_{dB}=20\cdot\log_{10}\left(\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}\right)$$

Let's consider three cases for the value of the frequency, and determine the magnitude in each case.:

Case 1) $\omega \ll \omega_0$. This is the low frequency case with $\omega/\omega_0 \rightarrow 0$. We can write an approximation for the magnitude of the transfer function:

$$\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}\approx 1,\text{ and }\left|H\left(j\omega\right)\right|_{dB}\approx 20\cdot\log_{10}\left(\frac{1}{1}\right)=0$$

This low frequency approximation is shown in blue on the diagram below.

Case 2) $\omega \gg \omega_0$. This is the high frequency case with $\omega/\omega_0 \rightarrow \infty$. We can write an approximation for the magnitude of the transfer function:

$$\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}\approx\sqrt{\left(\frac{\omega}{\omega_0}\right)^2}\approx\frac{\omega}{\omega_0},\text{ so}$$
$$\left|H\left(j\omega\right)\right|_{dB}\approx 20\cdot\log_{10}\left(\frac{\omega_0}{\omega}\right)$$

The high frequency approximation is at shown in green on the diagram below. It is a straight line with a slope of -20 dB/decade going through the break frequency at 0 dB (if $\omega=\omega_0$ the approximation simplifies to 0 dB; $\omega=10\cdot\omega_0$ gives an approximate gain of 0.1, or -20 dB and so on). That is, the approximation goes through 0 dB at $\omega=\omega_0$, and for every factor of 10 increase in frequency, the magnitude drops by 20 dB..

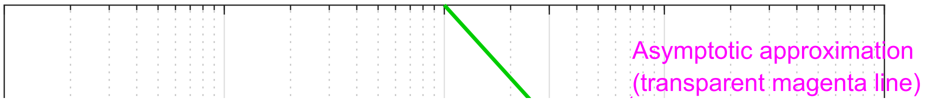
Case 3) $\omega=\omega_0$. At the break frequency

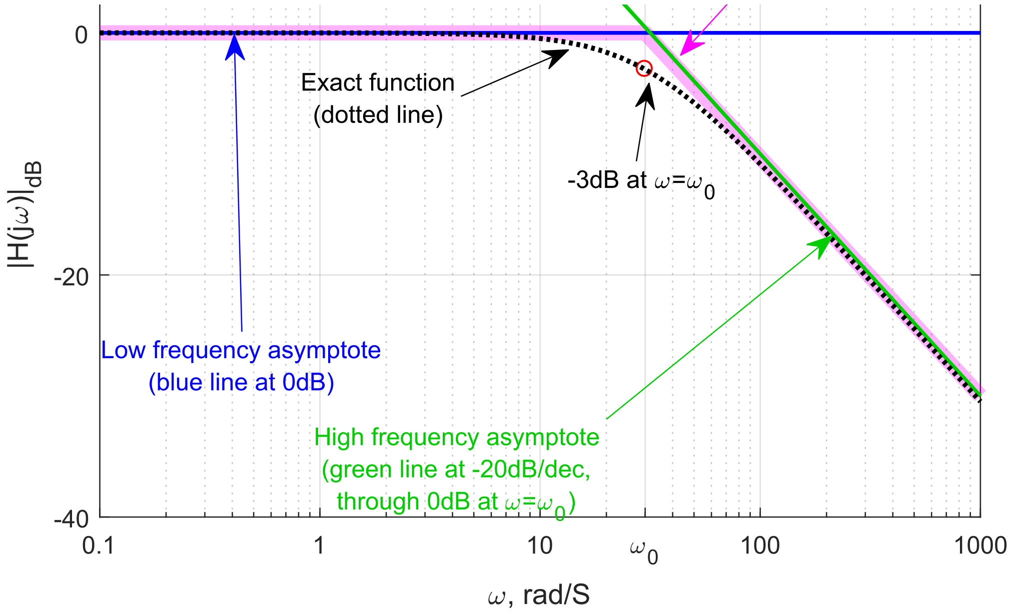
$$\left|H\left(j\omega_0\right)\right|_{dB}=20\cdot\log_{10}\left(\frac{1}{\sqrt{1+\left(\frac{\omega_0}{\omega_0}\right)^2}}\right)=20\cdot\log_{10}\left(\frac{1}{\sqrt{2}}\right)\approx -3\ dB$$

This point is shown as a red circle on the diagram.

To draw a piecewise linear approximation, use the low frequency asymptote up to the break frequency, and the high frequency asymptote thereafter.

Magnitude of Simple Real Pole





The resulting asymptotic approximation is shown highlighted in transparent magenta. The maximum error between the asymptotic approximation and the exact magnitude function occurs at the break frequency and is approximately -3 dB.

Magnitude of a real pole: The piecewise linear asymptotic Bode plot for magnitude is at 0 dB until the break frequency and then drops at 20 dB per decade as frequency increases (i.e., the slope is -20 dB/decade).

Phase

The phase of a single real pole is given by is given by

$$\angle H(j\omega) = \angle\left(\frac{1}{1+j\frac{\omega}{\omega_0}}\right) = -\angle\left(1+j\frac{\omega}{\omega_0}\right) = -\arctan\left(\frac{\omega}{\omega_0}\right)$$

Let us again consider three cases for the value of the frequency:

Case 1) $\omega \ll \omega_0$. This is the low frequency case with $\omega/\omega_0 \to 0$. At these frequencies We can write an approximation for the phase of the transfer function

$$\angle H(j\omega) \approx -\arctan(0) = 0^\circ = 0\ rad$$

The low frequency approximation is shown in blue on the diagram below.

Case 2) $\omega \gg \omega_0$. This is the high frequency case with $\omega/\omega_0 \to \infty$. We can write an approximation for the phase of the transfer function

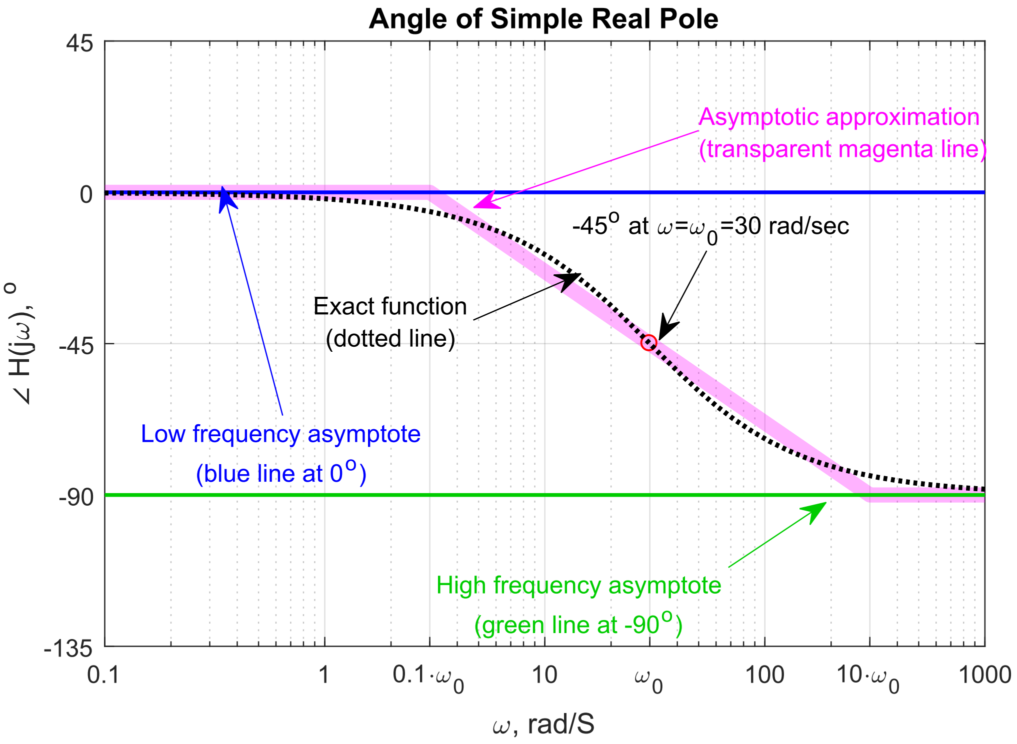
$$\angle H(j\omega) \approx -\arctan(\infty) = -90^\circ = -\frac{\pi}{2}\ rad$$

The high frequency approximation is at shown in green on the diagram below. It is a horizontal line at -90°.

Case 3) $\omega=\omega_0$. The break frequency. At this frequency

$$\angle H(j\omega) = -\arctan(1) = -45^\circ = -\frac{\pi}{4}\,rad$$

This point is shown as a red circle on the diagram.



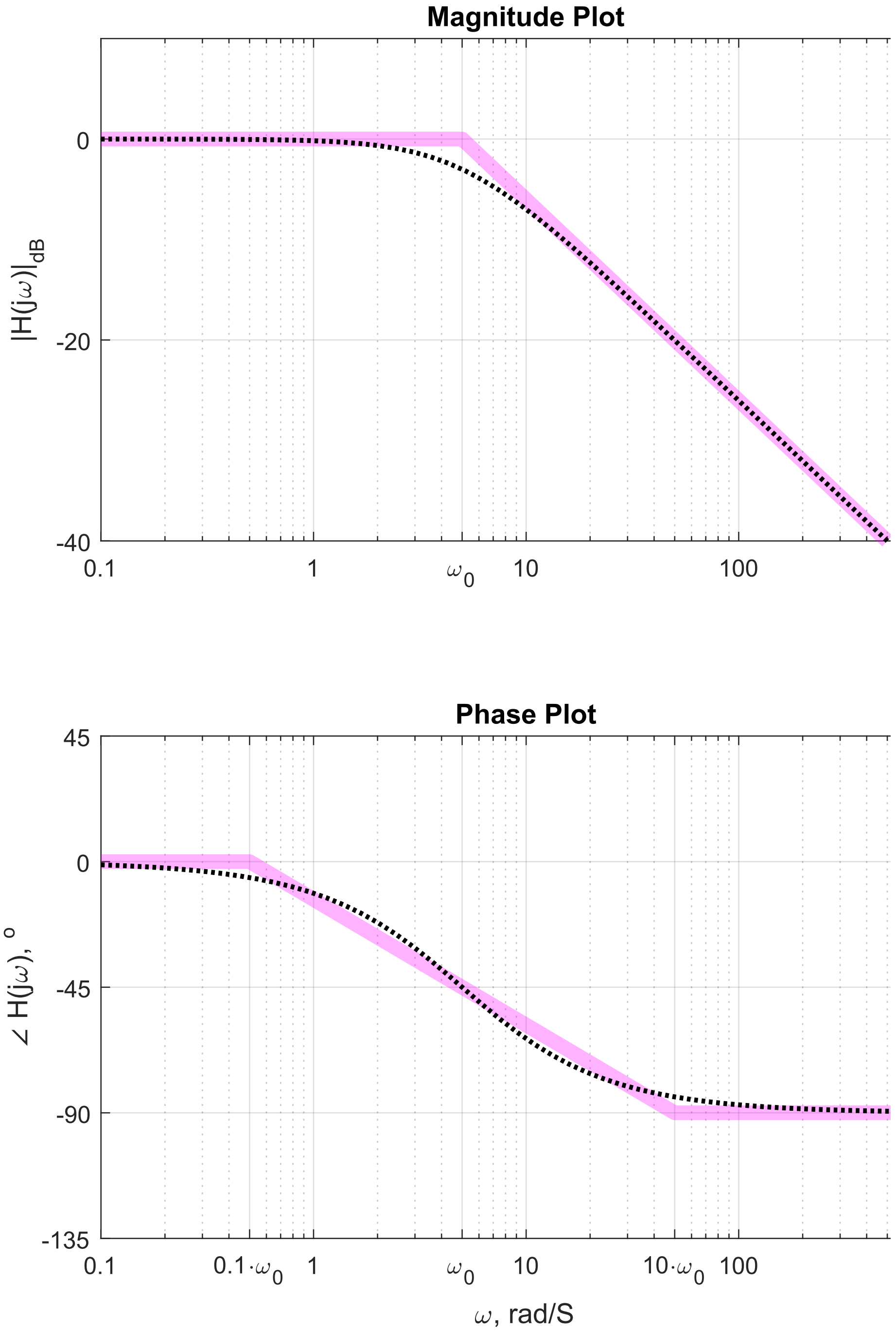
A piecewise linear approximation is not as easy in this case because the high and low frequency asymptotes don't intersect. Instead we use a rule that follows the exact function fairly closely, but is also somewhat arbitrary. Its main advantage is that it is easy to remember.

Phase of a real pole: The piecewise linear asymptotic Bode plot for phase follows the low frequency asymptote at 0° until one tenth the break frequency ($0.1\cdot\omega_0$) then decrease linearly to meet the high frequency asymptote at ten times the break frequency ($10\cdot\omega_0$). This line is shown above. Note that there is no error at the break frequency and about 5.7° of error at $0.1\cdot\omega_0$ and $10\cdot\omega_0$ the break frequency.

Example: Real Pole

The first example is a simple pole at 5 radians per second. The asymptotic approximation is magenta, the exact function is a dotted black line.

$$H(s) = \frac{1}{1+\frac{s}{5}}$$

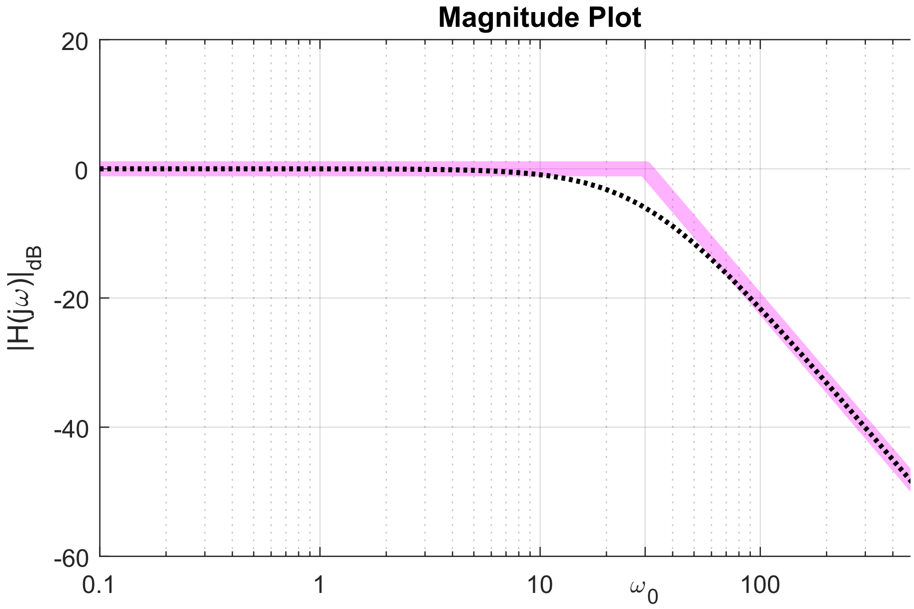


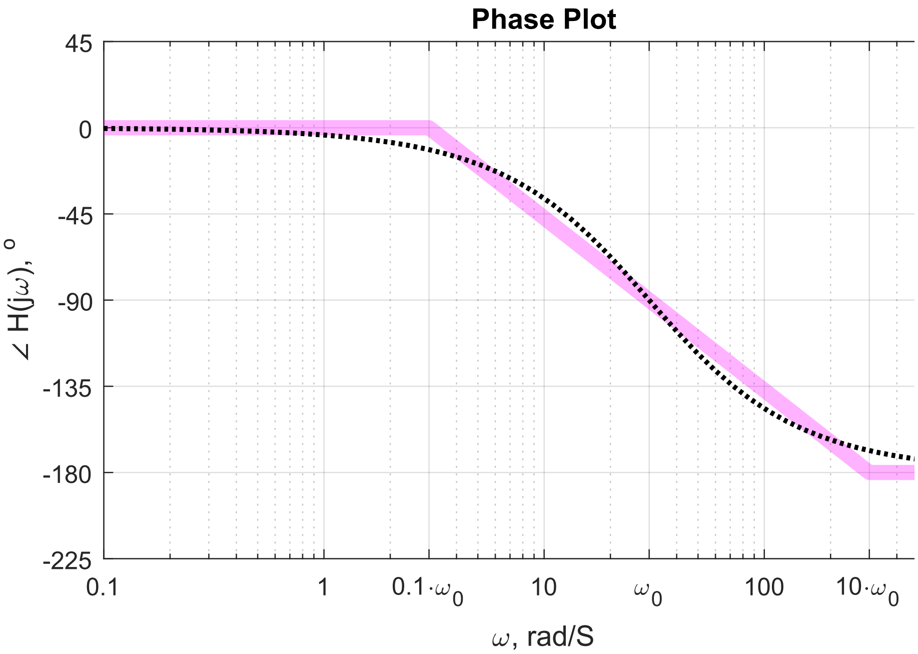
Example: Repeated Real Pole

The second example shows a double pole at 30 radians per second. Note that the slope of the asymptote is -40 dB/decade and the phase goes from 0 to -180°. The effect of repeating a pole is to double the slope of the magnitude to -40 dB/decade and the slope of the phase to -90°/decade.

$$H(s) = \frac{1}{\quad}$$

$$\left(1+\frac{s}{30}\right)^2$$





Key Concept: Bode Plot for Real Pole

- For a simple real pole the piecewise linear asymptotic Bode plot for magnitude is at 0 dB until the break frequency and then drops at 20 dB per decade (i.e., the slope is -20 dB/decade). An nth order pole has a slope of -20·n dB/decade.
- The phase plot is at 0° until one tenth the break frequency and then

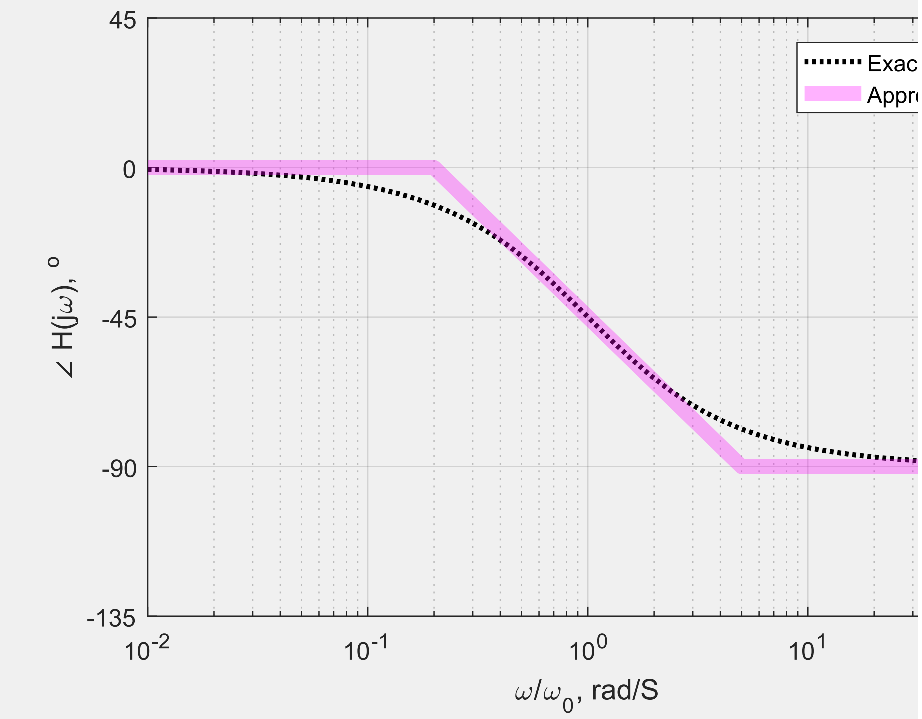
drops linearly to -90° at ten times the break frequency. An n^{th} order pole drops to -90°·n.

The analysis given above assumes ω_0 is positive. For negative ω_0 here.

Interactive Demo

Aside: a different formulation of the phase approximation

There is another approximation for phase that is occasionally used. The approximation is developed by matching the slope of the actual phase term to that of the approximation at $\omega=\omega_0$. Using math similar to that given here (for the underdamped case) it can be shown that by drawing a line starting at 0° at $\omega=\omega_0/e^{\pi/2}=\omega_0/4.81$ (or $\omega_0\cdot e^{-\pi/2}$) to -90° at $\omega=\omega_0\cdot4.81$ we get a line with the same slope as the actual function at $\omega=\omega_0$. The approximation described previously is much more commonly used as is easier to remember as a line drawn from 0° at $\omega_0/5$ to -90° at $\omega_0\cdot5$, and easier to draw on semi-log paper. The latter is shown on the diagram below.



Although this method is more accurate in the region around $\omega=\omega_0$ there is a larger maximum error (more than 10°) near $\omega_0/5$ and $\omega_0\cdot5$ when compared to the method described previously.

A Real Zero

The piecewise linear approximation for a zero is much like that for a pole

The piecewise linear approximation for a zero is much like that for a pole
Consider a simple zero: $H(s)=1+\frac{s}{\omega_0},\quad H(j\omega)=1+j\frac{\omega}{\omega_0}$.

Magnitude

The development of the magnitude plot for a zero follows that for a pole. Refer to the previous section for details. The magnitude of the zero is given by

$$\left|H\left(j\omega\right)\right|=\left|1+j\frac{\omega}{\omega_0}\right|$$

Again, as with the case of the real pole, there are three cases:

1. At low frequencies, $\omega<<\omega_0$, the gain is approximately 1 (or 0 dB).
2. At high frequencies, $\omega>>\omega_0$, the gain increases at 20 dB/decade and goes through the break frequency at 0 dB.
3. At the break frequency, $\omega=\omega_0$, the gain is about 3 dB.

Magnitude of a Real Zero: For a simple real zero the piecewise linear asymptotic Bode plot for magnitude is at 0 dB until the break frequency and then increases at 20 dB per decade (i.e., the slope is +20 dB/decade).

Phase

The phase of a simple zero is given by:

$$\angle H\left(j\omega\right)=\angle\left(1+j\frac{\omega}{\omega_0}\right)=\arctan\left(\frac{\omega}{\omega_0}\right)$$

The phase of a single real zero also has three cases (which can be derived similarly to those for the real pole, given above):

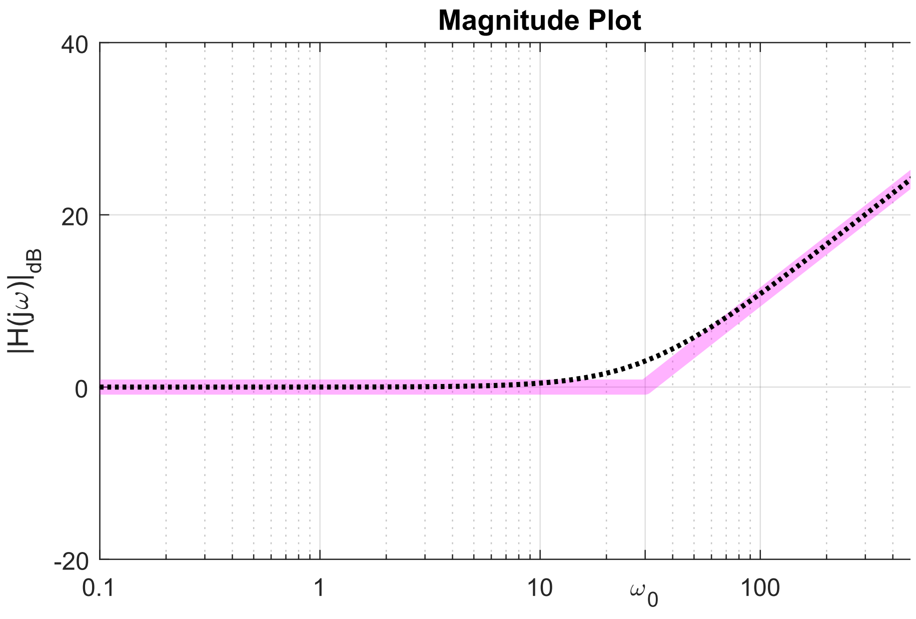
1. At low frequencies, $\omega<<\omega_0$, the phase is approximately zero.
2. At high frequencies, $\omega>>\omega_0$, the phase is +90°.
3. At the break frequency, $\omega=\omega_0$, the phase is +45°.

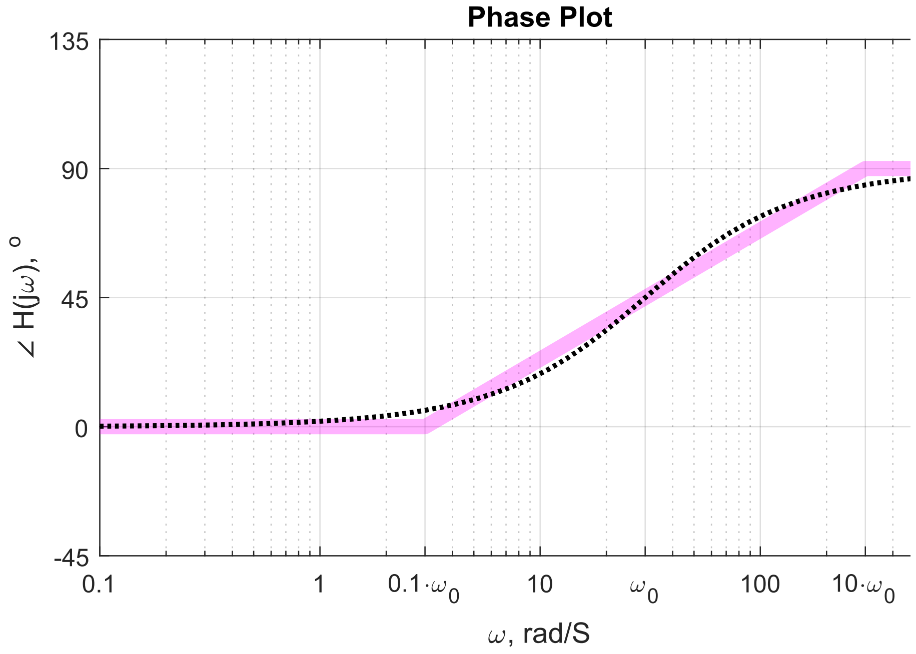
Phase of a Real Zero: Follow the low frequency asymptote at 0° until one tenth the break frequency (0.1 ω_0) then increase linearly to meet the high frequency asymptote at ten times the break frequency (10 ω_0).

Example: Real Zero

This example shows a simple zero at 30 radians per second. The asymptotic approximation is magenta, the exact function is the dotted black line.

$$H(s)=1+\frac{s}{30}$$





Key Concept: Bode Plot of Real Zero:

- The plots for a real zero are like those for the real pole but mirrored about 0dB or 0°.
- For a simple real *zero* the piecewise linear asymptotic Bode plot for magnitude is at 0 dB until the break frequency and then *rises* at +20 dB per decade (i.e., the slope is +20 dB/decade). An n th order *zero* has a slope of +20·n dB/decade.
- The phase plot is at 0° until one tenth the break frequency and then *rises* linearly to +90° at ten times the break frequency. An nth order

zero rises to +90°·n.

The analysis given above assumes the ω_0 *is positive. For negative* ω_0 *here.*

Interactive Demo

A Pole at the Origin

A pole at the origin is easily drawn exactly. Consider

$$H(s) = \frac{1}{s}, \quad H(j\omega) = \frac{1}{j\omega} = -\frac{j}{\omega}$$

Magnitude

The magnitude is given by

$$|H(j\omega)| = \left|-\frac{j}{\omega}\right| = \frac{1}{\omega}$$

$$|H(j\omega)|_{dB} = 20 \cdot \log_{10}\left(\frac{1}{\omega}\right) = -20 \cdot \log_{10}(\omega)$$

In this case there is no need for approximate functions and asymptotes, we can plot the exact funtion. The function is represented by a straight line on a Bode plot with a slope of -20 dB per decade and going through 0 dB at 1 rad/sec. It also goes through 20 dB at 0.1 rad/sec, -20 dB at 10 rad/sec... Since there are no parameters (i.e., ω_0) associated with this function, it is always drawn in exactly the same manner.

Magnitude of Pole at the Origin: Draw a line with a slope of -20 dB/decade that goes through 0 dB at 1 rad/sec.

Phase

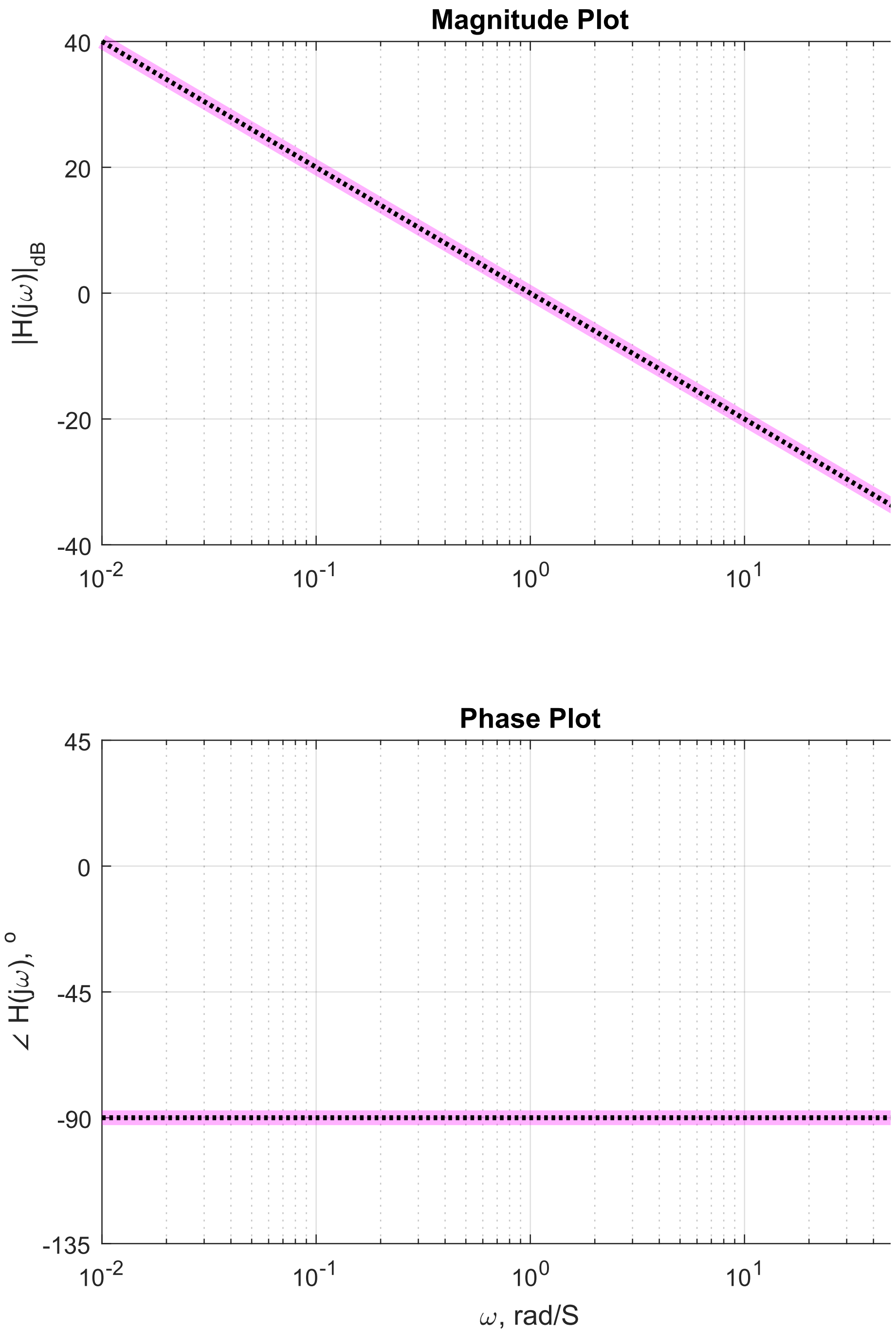
The phase of a simple zero is given by (H(jω) is a negative imaginary number for all values of ω so the phase is always -90°):

$$\angle H(j\omega) = \angle\left(-\frac{j}{\omega}\right) = -90^\circ$$

Phase of pole at the origin: The phase for a pole at the origin is -90°.

Example: Pole at Origin

This example shows a simple pole at the origin. The exact (dotted black line) is the same as the approximation (magenta).



Key Concept: Bode Plot for Pole at Origin

No interactive demo is provided because the plots are always drawn in the same way.

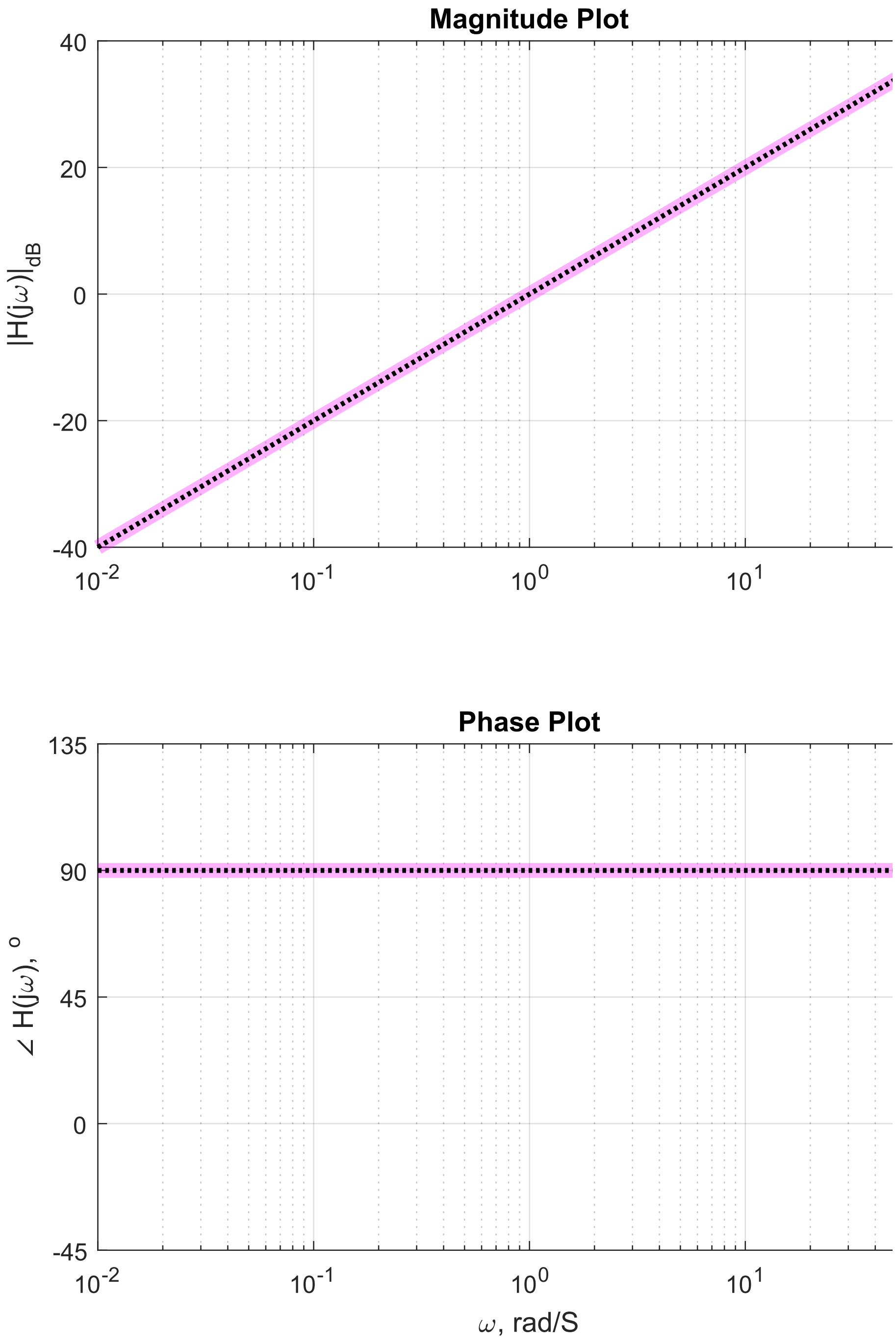
- For a simple pole at the origin draw a straight line with a slope of -20 dB per decade and going through 0 dB at 1 rad/ sec.
- The phase plot is at -90°.
- The magnitude of an nth order pole has a slope of -20·n dB/decade and a constant phase of -90°·n.

A Zero at the Origin

A zero at the origin is just like a pole at the origin but the magnitude increases with increasing ω, and the phase is +90° (i.e. simply mirror the graphs for the pole around the origin around 0dB or 0°).

Example: Zero at Origin

This example shows a simple zero at the origin. The exact (dotted black line) is the same as the approximation (magenta).



Key Concept: Bode Plot for Zero at Origin

- The plots for a zero at the origin are like those for the pole but mirrored about 0dB or 0°.
- For a simple zero at the origin draw a straight line with a slope of +20 dB per decade and going through 0 dB at 1 rad/ sec.
- The phase plot is at +90°.
- The magnitude of an nth order zero has a slope of +20·n dB/decade and a constant phase of +90°·n.

A Complex Conjugate Pair of Poles

The magnitude and phase plots of a complex conjugate (underdamped) pair of poles is more complicated than those for a simple pole. Consider the transfer function (with 0<ζ<1):

$$H(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right) + 1}$$

The analysis given below assumes the ζ is positive. For negative ζ see here.

Magnitude

The magnitude is given by

$$|H(j\omega)| = \left|\frac{1}{\left(\frac{j\omega}{\omega_0}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_0}\right) + 1}\right| = \left|\frac{1}{-\left(\frac{\omega}{\omega_0}\right)^2 + j2\zeta\left(\frac{\omega}{\omega_0}\right) + 1}\right| = \left|\frac{}{\left(1\right.}\right.$$

$$= \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_0}\right)^2}}$$

$$|H(j\omega)|_{dB} = -20 \cdot \log_{10}\left(\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_0}\right)^2}\right)$$

As before, let's consider three cases for the value of the frequency:

Case 1) $\omega \ll \omega_0$. This is the low frequency case. We can write an approximation for the magnitude of the transfer function

$$|H(j\omega)|_{dB} = -20\cdot\log_{10}(1) = 0$$

The low frequency approximation is shown in red on the diagram below.

Case 2) $\omega >> \omega_0$. This is the high frequency case. We can write an approximation for the magnitude of the transfer function

$$|H(j\omega)|_{dB} = -20\cdot\log_{10}\left(\left(\frac{\omega}{\omega_0}\right)^2\right) = -40\cdot\log_{10}\left(\frac{\omega}{\omega_0}\right)$$

The high frequency approximation is at shown in green on the diagram below. It is a straight line with a slope of -40 dB/decade going through the break frequency at 0 dB. That is, for every factor of 10 increase in frequency, the magnitude drops by 40 dB.

Case 3) $\omega \approx \omega_0$. It can be shown that a peak occurs in the magnitude plot near the break frequency. The derivation of the approximate amplitude and location of the peak are given here. We make the approximation that a peak exists only when

$$0<\zeta<0.5$$

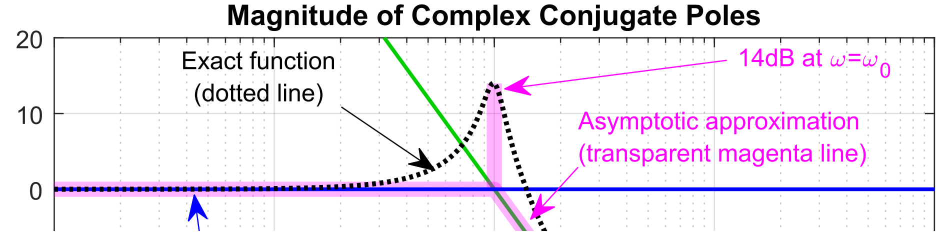
and that the peak occurs at ω_0 with height $1/(2\cdot\zeta)$.

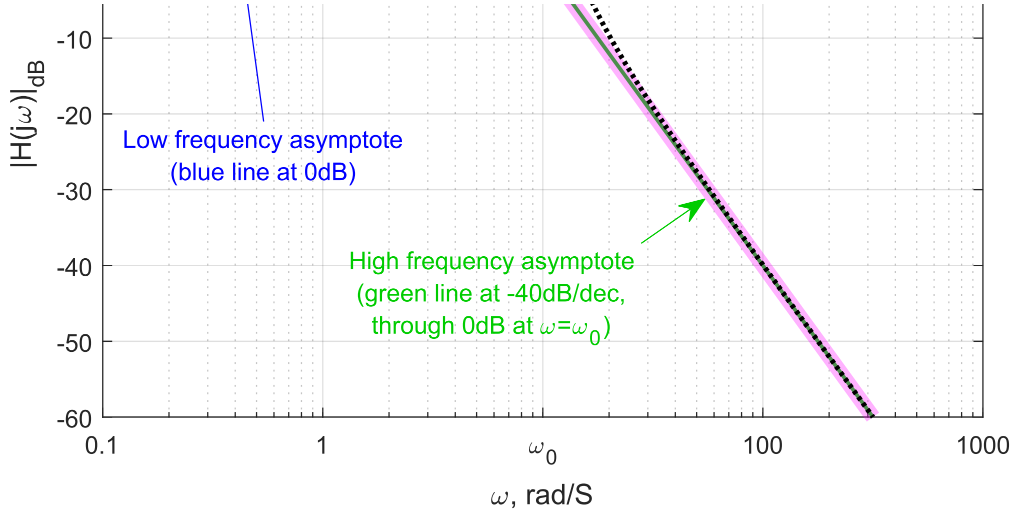
To draw a piecewise linear approximation, use the low frequency asymptote up to the break frequency, and the high frequency asymptote thereafter. If $\zeta<0.5$, then draw a peak of amplitude $1/(2\cdot\zeta)$ Draw a smooth curve between the low and high frequency asymptote that goes through the peak value.

As an example for the curve shown below $\omega_0=10$, $\zeta=0.1$,

$$H(s) = \frac{1}{\frac{s^2}{100} + 0.02\zeta s + 1} = \frac{1}{\left(\frac{s}{10}\right)^2 + 0.2\left(\frac{s}{10}\right) + 1} = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right) -}$$

The peak will have an amplitude of $1/(2\cdot\zeta)=5.00$ or 14 dB.





The resulting asymptotic approximation is shown as a black dotted line, the exact response is a black solid line.

Magnitude of Underdamped (Complex) poles: Draw a 0 dB at low frequencies until the break frequency, ω_0, and then drops with a slope of -40 dB/decade. If $\zeta<0.5$ we draw a peak of height at ω_0, otherwise no peak is drawn.

$$|H(j\omega_0)| \approx \frac{1}{2\zeta}, \quad |H(j\omega_0)|_{dB} \approx -20 \cdot \log_{10}(2\zeta)$$

Note: The actual height of the peak and its frequency are both slightly less than the approximations given above. An in depth discussion of the magnitude and phase approximations (along with some alternate approximations) are given here.

Phase

The phase of a complex conjugate pole is given by is given by

$$\angle H(j\omega) = \angle\left(\frac{1}{\left(\frac{j\omega}{\omega_0}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_0}\right) + 1}\right) = -\angle\left(\left(\frac{j\omega}{\omega_0}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_0}\right) + 1\right.$$

$$= -\arctan\left(\frac{2\zeta\frac{\omega}{\omega_0}}{1-\left(\frac{\omega}{\omega_0}\right)^2}\right)$$

Let us again consider three cases for the value of the frequency:

Case 1) $\omega<<\omega_0$. This is the low frequency case. At these frequencies We can write an approximation for the phase of the transfer function

$$\angle H(j\omega) \approx -\arctan\left(\frac{2\zeta\omega}{\;}\right) \approx -\arctan(0) = 0^\circ = 0\ rad$$

$$\left(\omega_0 \right)$$

The low frequency approximation is shown in red on the diagram below.

Case 2) ω>>ω$_0$. This is the high frequency case. We can write an approximation for the phase of the transfer function

$$\angle H\left(j\omega\right) \approx -180^\circ = -\pi\ rad$$

Note: this result makes use of the fact that the arctan function returns a result in quadrant 2 since the imaginary part of H(jω) is negative and the real part is positive.

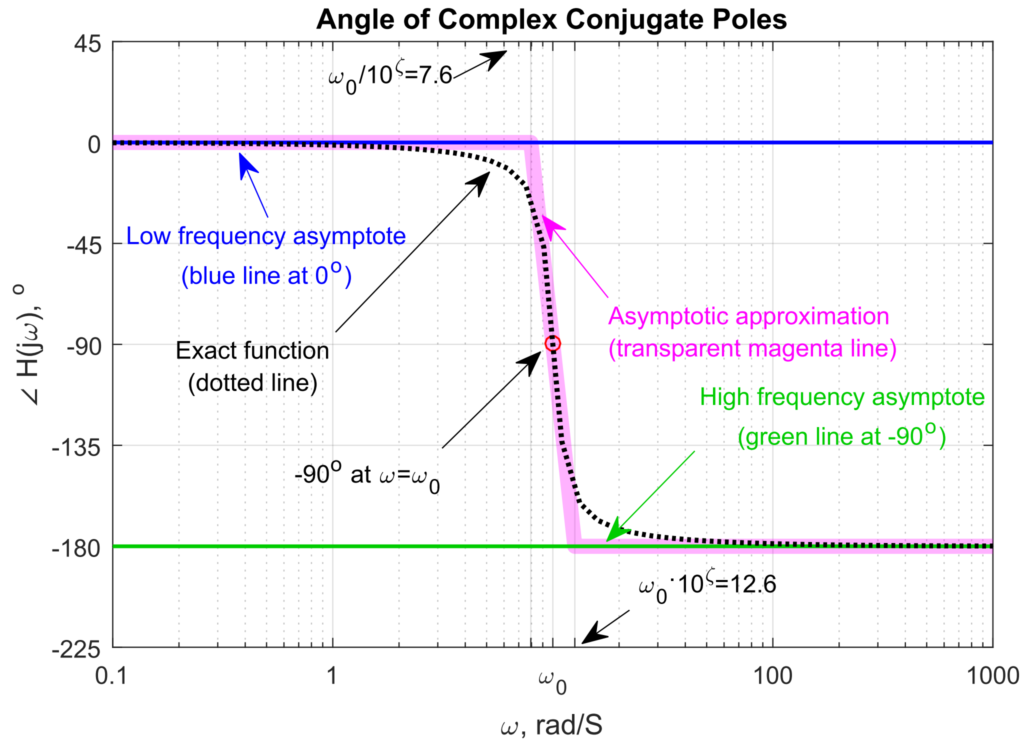
The high frequency approximation is at shown in green on the diagram below. It is a straight line at -180°.

Case 3) ω=ω$_0$. The break frequency. At this frequency

$$\angle H(j\omega_0) = -90^\circ$$

The asymptotic approximation is shown below for ω$_0$=10, ζ=0.1, followed by an explanation

$$H(s) = \frac{1}{\frac{s^2}{100} + 0.02\zeta s + 1} = \frac{1}{\left(\frac{s}{10}\right)^2 + 0.2\left(\frac{s}{10}\right) + 1} = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right) -}$$



A piecewise linear approximation is a bit more complicated in this case, and there are no hard and fast rules for drawing it. The most common way is to look up a graph in a textbook with a chart that shows phase plots for many values of ζ. Three asymptotic approximations are given here. We will use the approximation that connects the the low frequency asymptote to the high frequency asymptote starting at

$$\omega = \frac{\omega_0}{10^{\zeta}} = \omega_0 \cdot 10^{-\zeta}$$

and ending at

$$\omega = \omega_0 \cdot 10^{\zeta}$$

Since ζ=0.2 in this case this means that the phase starts at 0° and then breaks downward at $\omega=\omega_0/10^{\zeta}$=7.9 rad/sec. The phase reaches -180° at $\omega=\omega_0\cdot 10^{\zeta}$=12.6 rad/sec.

As a practical matter If ζ<0.02, the approximation can be simply a vertical line at the break frequency. One advantage of this approximation is that it is very easy to plot on semilog paper. Since the number $10\cdot\omega_0$ moves up by a full decade from ω_0, the number $10^{\zeta}\cdot\omega_0$ will be a fraction ζ of a decade above ω_0. For the example above the corner frequencies for ζ=0.1 fall near ω_0 one tenth of the way between ω_0 and $\omega_0/10$ (at the lower break frequency) to one tenth of the way between ω_0 and $\omega_0\cdot 10$ (at the higher frequency).

Phase of Underdamped (Complex) Poles: Follow the low frequency asymptote at 0° *until*

$$\omega = \frac{\omega_0}{10^{\zeta}}$$

then decrease linearly to meet the high frequency asymptote at -180° at

$$\omega = \omega_0 \cdot 10^{\zeta}$$

Other magnitude and phase approximations (along with exact expressions) are given here.

Key Concept: Bode Plot for Complex Conjugate Poles

- For the magnitude plot of complex conjugate poles draw a 0 dB at low frequencies, go through a peak of height,

$$|H(j\omega_0)| \approx \frac{1}{2\zeta}, \quad |H(j\omega_0)|_{dB} \approx -20 \cdot \log_{10}(2\zeta)$$

at the break frequency and then drop at 40 dB per decade (i.e., the slope is -40 dB/decade). The high frequency asymptote goes through the break frequency. Note that in this approximation the peak only exists for

$$0 < \zeta < 0.5$$

- To draw the phase plot simply follow low frequency asymptote at 0° until

$$\omega = \frac{\omega_0}{10^\zeta} = \omega_0 \cdot 10^{-\zeta}$$

then decrease linearly to meet the high frequency asymptote at -180° at

$$\omega = \omega_0 \cdot 10^\zeta$$

If $\zeta<0.02$, the approximation can be simply a vertical line at the break frequency.

- Note that the *shape* of the graphs (magnitude peak height, steepness of phase transition) are determined solely by ζ, and the frequency at which the magnitude peak and phase transition occur are determined solely by ω_0.

Note: Other magnitude and phase approximations (along with exact expressions) are given here.
The analysis given above assumes the ζ is positive. For negative ζ see here

Interactive Demo

A Complex Conjugate Pair of Zeros

Not surprisingly a complex pair of zeros yields results similar to that for a complex pair of poles. The magnitude and phase plots for the complex zero are the mirror image (around 0dB for magnitude and around 0° for phase) of those for the complex pole. Therefore, the magnitude has a dip instead of a peak, the magnitude increases above the break frequency and the phase increases rather than decreasing. The results will not be derived here, but closely follow those for complex poles.

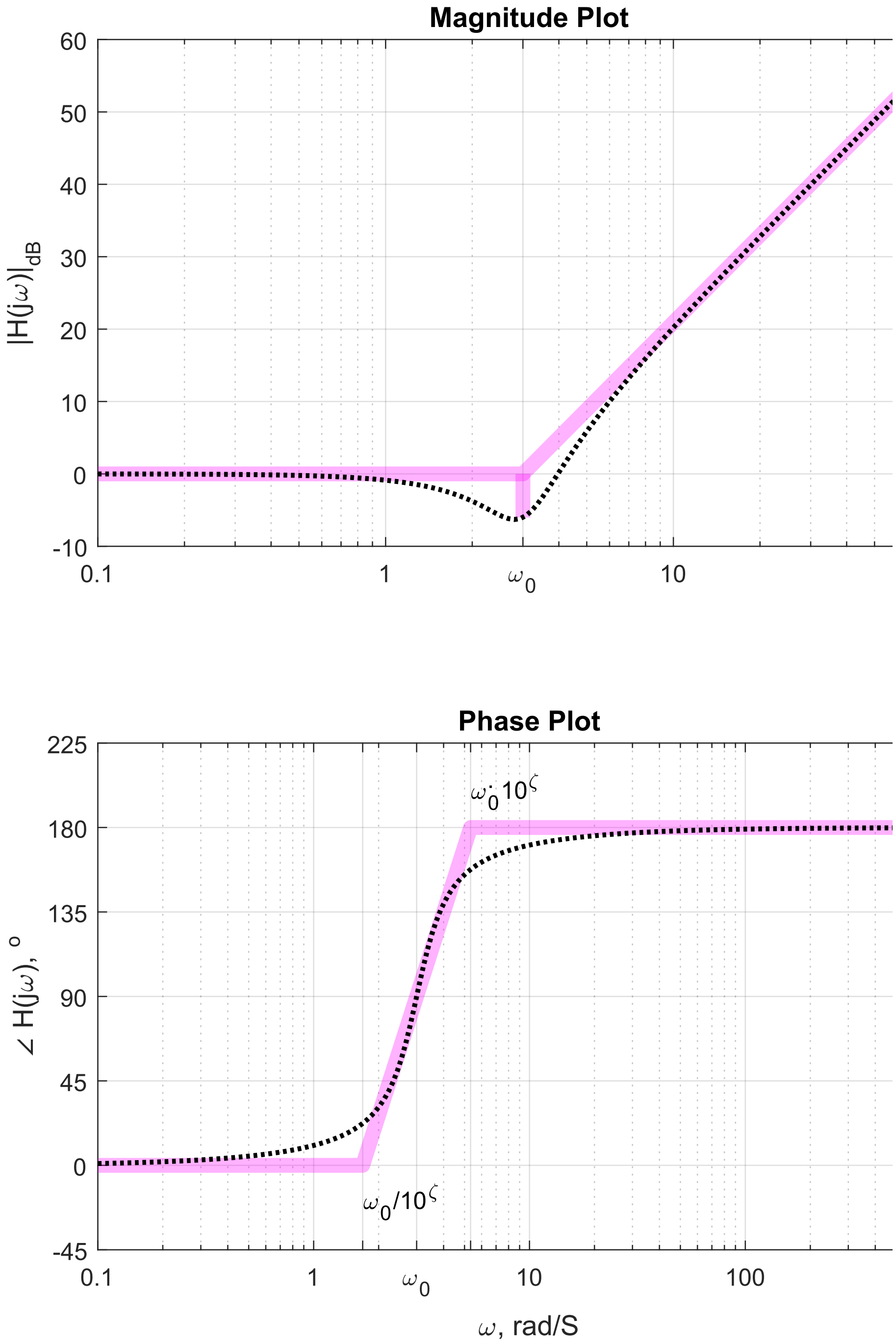
Note: The analysis given below assumes the ζ is positive. For negative ζ see here

Example: Complex Conjugate Zero

The graph below corresponds to a complex conjugate zero with ω_0=3, ζ=0.25

$$H(s) = \left(\frac{s}{\omega_0}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right) + 1$$

The dip in the magnitude plot will have a magnitude of 0.5 or -6 dB. The break frequencies for the phase are at $\omega=\omega_0/10^{\zeta}$=1.7 rad/sec and $\omega=\omega_0\cdot10^{\zeta}$=5.3 rad/sec.



Key Concept: Bode Plot of Complex Conjugate Zeros

- The plots for a complex conjugate pair of zeros are very much like those for the poles but mirrored about 0dB or 0°.
- For the magnitude plot of complex conjugate zeros draw a 0 dB at

- For the magnitude plot of complex conjugate zeros draw a 0 dB at low frequencies, go through a dip of magnitude:

  $$|H(j\omega_0)| \approx 2\zeta, \quad |H(j\omega_0)|_{dB} \approx 20 \cdot \log_{10}(2\zeta)$$

  at the break frequency and then rise at +40 dB per decade (i.e., the slope is +40 dB/decade). The high frequency asymptote goes through the break frequency. Note that the peak only exists for

  $$0 < \zeta < 0.5$$

- To draw the phase plot simply follow low frequency asymptote at 0° until

  $$\omega = \frac{\omega_0}{10^{\zeta}} = \omega_0 \cdot 10^{-\zeta}$$

  then increase linearly to meet the high frequency asymptote at 180° at

  $$\omega = \omega_0 \cdot 10^{\zeta}$$

- Note that the *shape* of the graphs (magnitude peak height, steepness of phase transition) are determined solely by ζ, and the frequency at which the magnitude peak and phase transition occur are determined solely by ω_0.

Note: Other magnitude and phase approximations (along with exact expressions) are given here.
The analysis given below assumes the ζ is positive. For negative ζ see here.

Interactive Demo

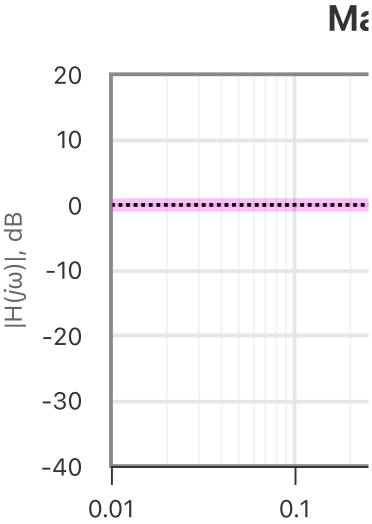
Non-Minimum Phase Systems

All of the examples above are for minimum phase systems. These systems have poles and zeros that do not have positive real parts. For example the term (s+2) is zero when s=-2, so it has a negative real root. First order poles and zeros have negative real roots if ω_0 is positive. Second order poles and zeros have negative real roots if ζ is positive. The magnitude plots for these systems remain unchanged, but the phase plots are inverted. See here for discussion.

Interactive Demos:

Below you will find interactive demos that show how to draw the asymptotic approximation for a constant, a first order pole and zero, and a second order (underdamped) pole and zero. Note there is no demo for a pole or zero at the origin because these are always drawn in exactly the same way; there are no variable parameters (i.e., ω_0 or ζ).

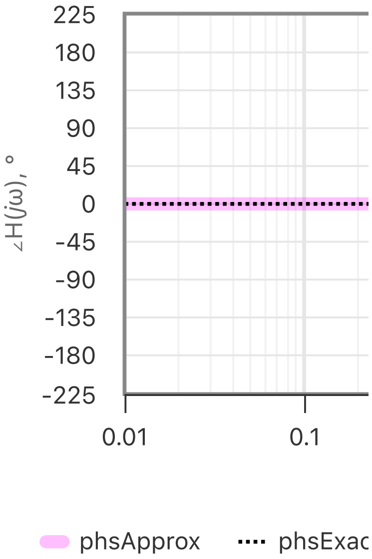
Interactive Demo: Bode Plot of Constant Term



This demonstration shows how the gain term affects a Bode plot. To run the demonstration either enter the value of K, or |K| expressed in dB, in one of the text boxes below. If you enter |K| in dB, then the sign of K is unchanged from its current value. You can also set |K| and ∠K by either clicking and dragging the horizontal lines on the graphs themselve. The magnitude of K must be between 0.01 and 100 (-40dB and +40dB). The phase of K (∠K) can only be 0° (for a positive value of K) or ±180° (for negative K). Enter a value for gain, K: 1.00 ,

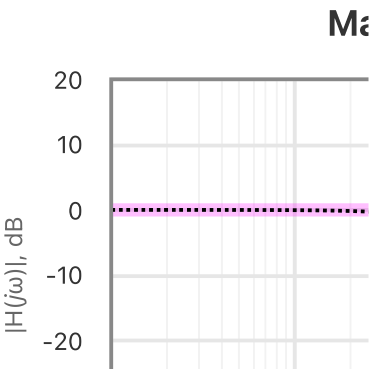
or enter |K| expressed in dB: 0.00 dB.

K = 1.00 so the value of $K_{dB} = 20\cdot\log_{10}(|K|) = 20\cdot\log_{10}(|1.00|) = 0.00$. Or, given that $K_{dB} = 0.00$, |K| $= 10^{Kdb/20} = 10^{0.00/20} = 1$. The sign of K depends on phase, in this case K is positive.and phase = 0°.



Note that for the case of a constant term, the approximate (magenta line) and exact (dotted black line) representations of magnitude and phase are equal.

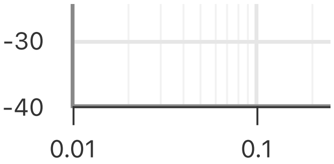
Interactive Demo: Bode Plot of a Real Pole



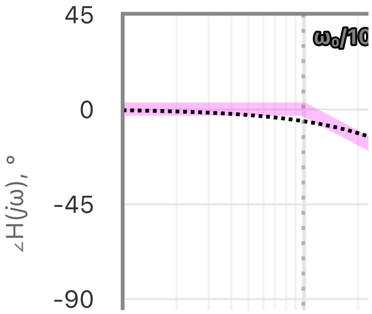
This demonstration shows how a first order pole expressed as:

$$H(s) = \frac{1}{1+\frac{s}{\omega_0}} = \frac{1}{1+j\frac{\omega}{\omega_0}},$$

is displayed on a Bode plot. To change the value of ω_0, you can either change the value in the text box, below, or drag the vertical line showing ω_0 on the graphs to the right. The exact values of magnitude and phase are shown as black dotted lines and the asymptotic approximations are shown with a thick magenta line. The value of ω_0 is constrained such that $0.1 \le \omega_0 \le 10$ rad/second.



Enter a value for ω_0: 1.000



Asymptotic Magnitude: The asymptotic magnitude plot starts (at low frequencies) at 0 dB and stays at that level until it gets to ω_0. At that point the gain starts dropping with a slope of

---

Rules for Constructing Bode Diagrams

Overview | Freq Domain | Asymptotic plots | Making Plot | Examples | Drawing Tool | BodePlotGui | Rules Table | Printable

This document will discuss how to actually draw Bode diagrams. It consists mostly of examples.

Key Concept -
To draw Bode diagram there are four steps:

1. Rewrite the transfer function in proper form.
2. Separate the transfer function into its constituent parts.
3. Draw the Bode diagram for each part.
4. Draw the overall Bode diagram by adding up the results from part 3.

1. Rewrite the transfer function in proper form.

A transfer function is normally of the form:

$$H(s) = K\frac{\sum\limits_{i=0}^{m} b_m s^m}{\sum\limits_{j=0}^{n} a_n s^n}$$

As discussed in the previous document, we would like to rewrite this so the lowest order term in the numerator and denominator are both unity.

Some examples will clarify:

Example 1

$$H(s) = 30\frac{s+10}{s^2+3s+50} = 30\frac{10}{50}\frac{\frac{s}{10}+1}{\frac{s^2}{50}+\frac{3}{50}s+1} = 6\frac{\frac{s}{10}+1}{\frac{s^2}{50}+\frac{3}{50}s+1}$$

Note that the final result has the lowest (zero) order power of numerator and denominator polynomial equal to unity.

Example 2

$$H(s) = 30\frac{5s}{s^2+3s+50} = 30\frac{5}{50}\frac{\frac{s}{1}}{\frac{s^2}{50}+\frac{3}{50}s+1} = 3\frac{\frac{s}{1}}{\frac{s^2}{50}+\frac{3}{50}s+1}$$

Note that in this example, the lowest power in the numerator was 1.

Example 3

$$H(s) = 30\frac{s+10}{(s+3)(s+50)} = 30\frac{10}{3\cdot 50}\frac{\frac{s}{10}+1}{\left(\frac{s}{3}+1\right)\left(\frac{s}{50}+1\right)}$$

$$= 2\frac{\frac{s}{10}+1}{\left(\frac{s}{3}+1\right)\left(\frac{s}{50}+1\right)}$$

In this example the denominator was already factored. In cases like this, each factored term needs to have unity as the lowest order power of s (zero in this case).

---

2. Separate the transfer function into its constituent parts.

The next step is to split up the function into its constituent parts. There are seven types of parts:

1. A constant
2. Poles at the origin
3. Zeros at the origin
4. Real Poles
5. Real Zeros
6. Complex conjugate poles
7. Complex conjugate zeros

We can use the examples above to demonstrate again.

Example 1

$$H(s) = 30\frac{s+10}{s^2+3s+50} = 30\frac{10}{50}\frac{\frac{s}{10}+1}{\frac{s^2}{50}+\frac{3}{50}s+1} = 6\frac{\frac{s}{10}+1}{\frac{s^2}{50}+\frac{3}{50}s+1}$$

This function has

- a constant of 6,
- a zero at s=-10,
- and complex conjugate poles at the roots of $s^2+3s+50$.

The complex conjugate poles are at s=-1.5 ± *j*6.9 (where *j*=sqrt(-1)). A more common (and useful for our purposes) way to express this is to use the standard notation for a second order polynomial

$$\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right)+1$$

In this case

$$\omega_0 = \sqrt{50} = 7.07, \quad \zeta = \frac{3\sqrt{50}}{2\cdot 50} = 0.21$$

Example 2

$$H(s) = 30\frac{5s}{s^2+3s+50} = 30\frac{5}{50}\frac{\frac{s}{1}}{\frac{s^2}{50}+\frac{3}{50}s+1} = 3\frac{\frac{s}{1}}{\frac{s^2}{50}+\frac{3}{50}s+1}$$

This function has

- a constant of 3,
- a zero at the origin,
- and complex conjugate poles at the roots of $s^2+3s+50$, in other words

$$\omega_0 = \sqrt{50} = 7.07, \quad \zeta = \frac{3\sqrt{50}}{2\cdot 50} = 0.21$$

Example 3

$$H(s) = 30\frac{s+10}{(s+3)(s+50)} = 2\frac{\frac{s}{10}+1}{\left(\frac{s}{3}+1\right)\left(\frac{s}{50}+1\right)}$$

This function has

- a constant of 2,
- a zero at s=-10, and
- poles at s=-3 and s=-50.

3. Draw the Bode diagram for each part.

The rules for drawing the Bode diagram for each part are summarized on a separate page. Examples of each are given later.

---

4. Draw the overall Bode diagram by adding up the results from step 3.

After the individual terms are drawn, it is a simple matter to add them together. See examples, below.

Examples: Draw Bode Diagrams for the following transfer functions

These examples are compiled on the next page.

Example 1

A simple pole

$$H(s)=\frac{100}{s+30}$$

Full Solution

Example 2

Multiple poles and zeros

$$H(s)=100\frac{(s+1)}{(s+10)(s+100)}=100\frac{(s+1)}{s^2+110s+1000}$$

Full Solution

Example 3

A pole at the origin and poles and zeros

$$H(s)=10\frac{s+10}{s^2+3s}$$

Full Solution

Example 4

Repeated poles, a zero at the origin, and a negative constant

$$H(s)=-100\frac{s}{s^3+12s^2+21s+10}$$

Full Solution

Example 5

Complex conjugate poles

$$H(s) = 30\frac{s+10}{s^2+3s+50}$$

Full Solution

Bode Plot Examples

Overview | Freq Domain | Asymptotic plots | Making Plot | Examples | Drawing Tool
BodePlotGui | Rules Table | Printable

Several examples of the construction of Bode plots are included here; click on the transfer function in the table below to jump to that example.

Examples (Click on Transfer Function)

1	2	3	4	5	
$\frac{100}{s+30}$ (a real pole)	$100\frac{s+1}{s^2+110s+1000}$ (real poles and zeros)	$10\frac{s+10}{s^2+3s}$ (pole at origin)	$-100\frac{s}{s^3+12s^2+21s+10}$ (repeated real poles, negative constant)	$30\frac{s+10}{s^2+3s+50}$ (complex conj. poles)	$4\frac{s}{s}$ (p c co

References

Rules for Drawing Bode Diagrams

Overview | Freq Domain | Asymptotic plots | Making Plot | Examples | Drawing Tool
BodePlotGui | Summary | Printable

The table below summarizes what to do for each type of term in a Bode Plot. This is also available as a Word Document or PDF.
The table assumes $\omega_0>0$. If $\omega_0<0$, magnitude is unchanged, but phase is reversed.

Term	Magnitude	Phase

Constant: K	20log$_{10}$(|K|)	K>0: 0° K<0: ±180
Pole at Origin (Integrator) $\frac{1}{s}$	-20 dB/decade passing through 0 dB at ω=1	-90°
Zero at Origin (Differentiator) s	+20 dB/decade passing through 0 dB at ω=1 *(Mirror image, around x axis,of Integrator)*	+90° *(Mirror image, ar* *axis, of Integrator*
Real Pole $\frac{1}{\frac{s}{\omega_0}+1}$	1. Draw low frequency asymptote at 0 dB. 2. Draw high frequency asymptote at -20 dB/decade. 3. Connect lines at ω_0.	1. Draw low freq asymptote at 2. Draw high fre asymptote at 3. Connect with straight line fr 0.1·ω_0 to 10·
Real Zero $\frac{s}{\omega_0}+1$	1. Draw low frequency asymptote at 0 dB. 2. Draw high frequency asymptote at +20 dB/decade. 3. Connect lines at ω_0. *(Mirror image, around x-axis, of Real Pole)*	1. Draw low freq asymptote at 2. Draw high fre asymptote at 3. Connect with straight line fr 0.1·ω_0 to 10· *(Mirror image, ar* *axis, of Real Pole*
Underdamped Poles (Complex conjugate poles) $\frac{1}{\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1}$ $0<\zeta<1$	1. Draw low frequency asymptote at 0 dB. 2. Draw high frequency asymptote at -40 dB/decade. 3. Connect lines at ω_0. 4. If ζ<0.5, then draw peak at ω_0 with amplitude |H(jω_0)|=-20·log$_{10}$(2ζ), else don't draw peak (it is very small).	1. Draw low freq asymptote at 2. Draw high fre asymptote at 3. Connect with line from $\omega = \frac{\omega_0}{10^\zeta}$ to *You can also look* *textbook for exam*
Underdamped Zeros (Complex conjugate zeros) $\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1$	1. Draw low frequency asymptote at 0 dB. 2. Draw high frequency asymptote at +40 dB/decade. 3. Connect lines at ω_0. 4. If ζ<0.5, then draw peak at ω_0 with amplitude |H(jω_0)|=+20·log$_{10}$(2ζ),	1. Draw low freq asymptote at 2. Draw high fre asymptote at 3. Connect with line from $\omega = \frac{\omega_0}{10^\zeta}$ to

$0 < \zeta < 1$	else don't draw peak (it is very small). *(Mirror image, around x-axis, of Underdamped Pole)*	*You can also look textbook for exam* *(Mirror image, ar axis, of Underda Pole)*

For multiple order poles and zeros, simply multiply the slope of the

BodePlotGui: A Tool for Generating Asymptotic Bode Diagrams

Overview | Freq Domain | Asymptotic plots | Making Plot | Examples | Drawing Tool | BodePlotGui | Rules Table | Printable

BodePlotGui is a graphical user interface written in the MATLAB® programming language. It takes a transfer function and splits it into its constituent elements, then draws the piecewise linear asymptotic approximation for each element. It is hoped that the BodePlotGui program will be a versatile program for teaching and learning the construction of Bode diagrams from piecewise linear approximations.

Files for the program are found here.

Note: the MATLAB GUI doesn't display well on all devices (some elements of the GUI may not show up). If you have this problem, simply run the MATLAB command "`guide`" and open the file*BodePlotGui.fig*. You can edit the size and layout of the GUI for your machine. Save it, and then rerun the *BodePlotGui.m* file.

I have stopped working on BodePlotGui and have developed a similar tool in JavaScript to make it more accessible (see the "Drawing Tool" tab, above). While MATLAB is extremely powerful, it is also very expensive.

Use of program.

A Simple Example.

Consider the transfer function:

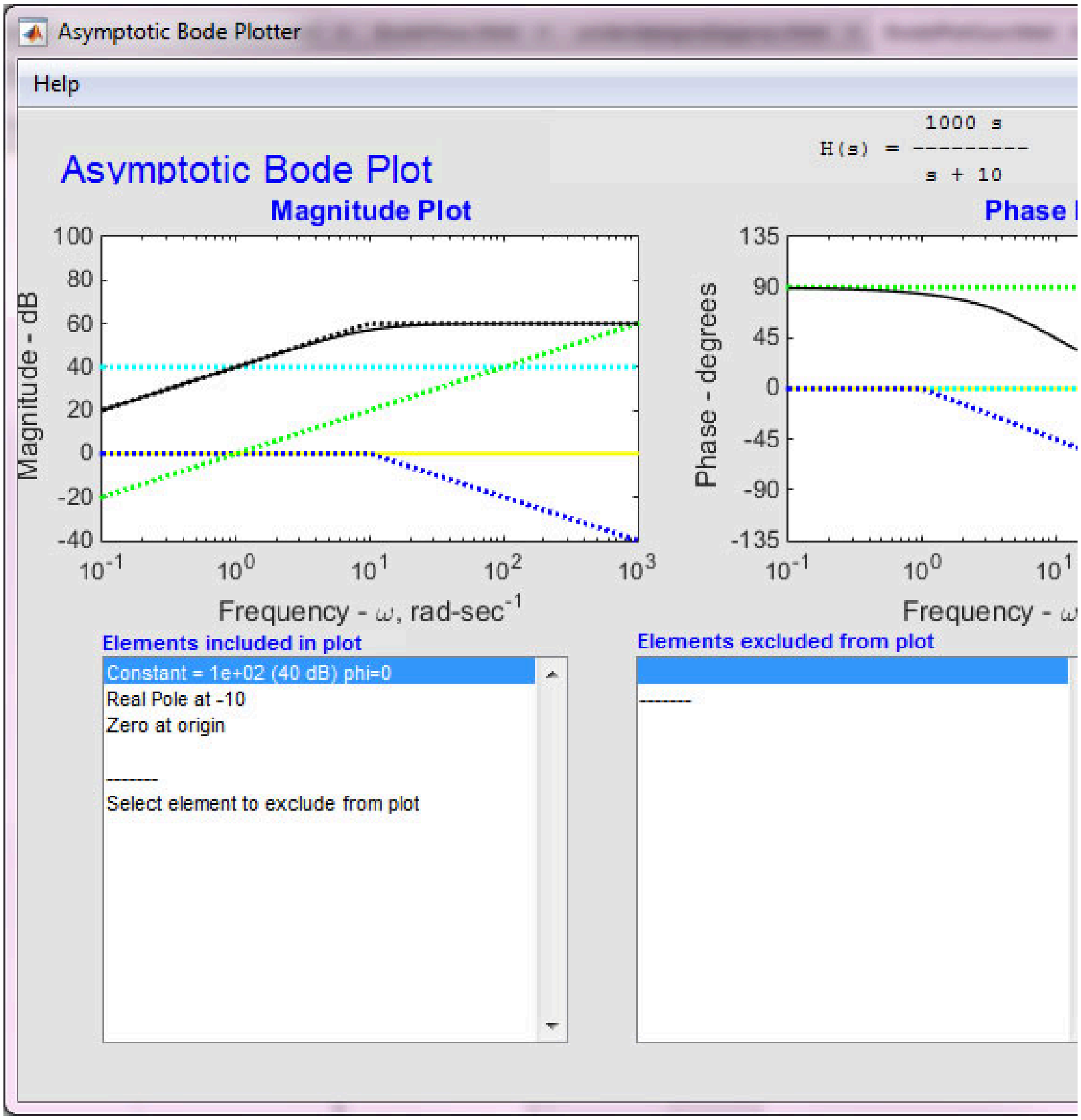
$$H(s) = 1000\frac{s}{s+10} = 100\frac{s}{1+\frac{s}{10}}$$

This function has three terms to be considered when constructing a Bode diagram, a constant (100), a pole at ω=10 rad/sec, and a zero at the origin. The following MATLAB® commands begin execution of the GUI:

```
>>MySys=tf(1000*[1 0],[1 10]);   %define Xfer functio
```

```
>>BodePlotGui(MySys)          %Invoke GUI
```

The GUI generates a window as shown below.



```
           1000 s
H(s) = ---------
          s + 10
```

Constant = 1e+02 (40 dB) phi=0
Real Pole at -10
Zero at origin

Select element to exclude from plot

Starting in the upper left and going counterclockwise, the windows show:

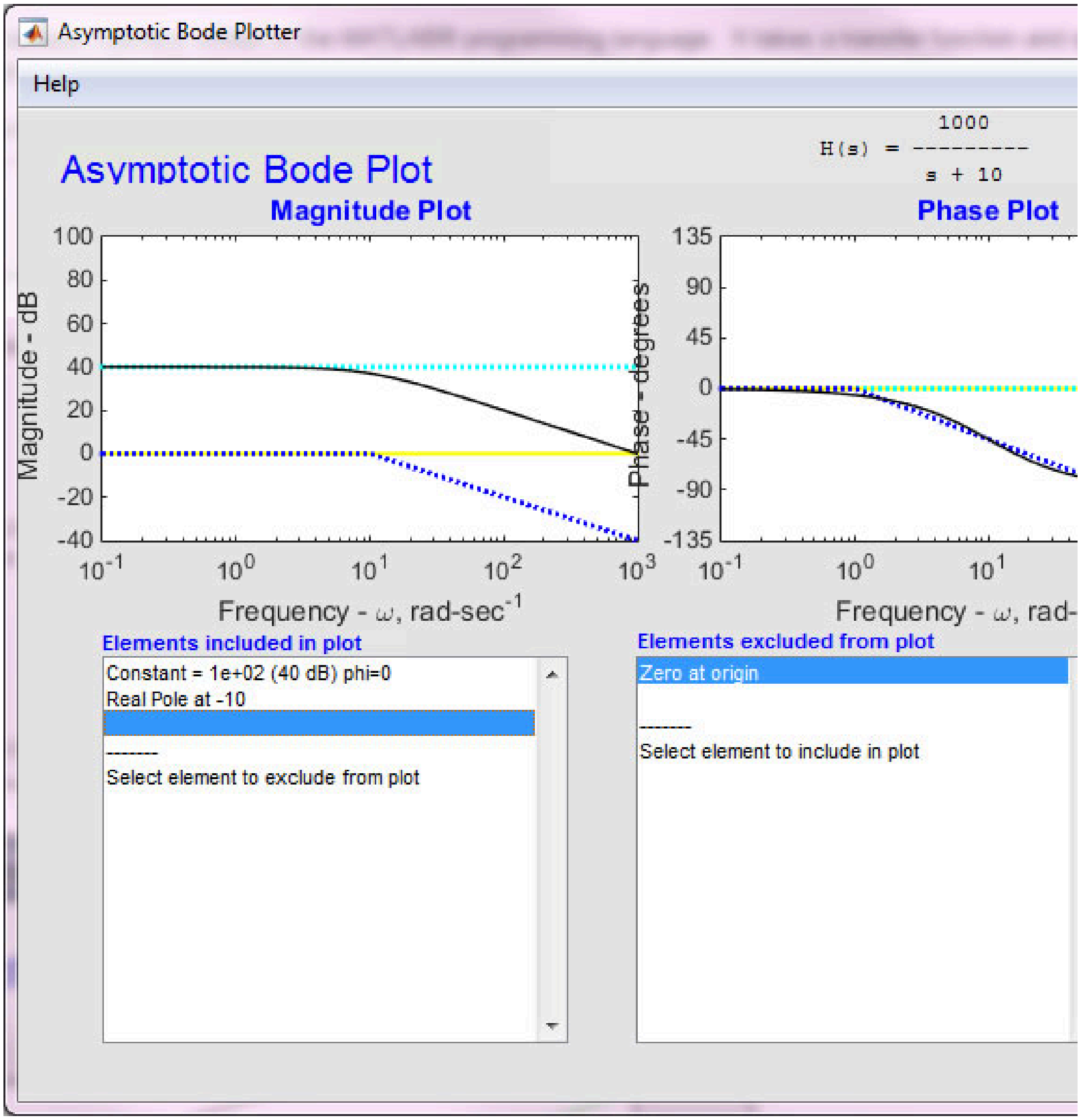
1. The magnitude plot, both the piecewise linear approximation for all three terms as well as the asymptotic plot for the complete transfer function and the exact Bode diagram for magnitude. Also shown is a zero reference line.
2. The phase plot.
3. A list of the systems in the user workspace.
4. Several checkboxes that let the user format the image. In particular there is a check-box that determines whether or not to display the asymptotic plot for the complete transfer function; sometimes it gets in the way of seeing the other plots, so you may want to hide it.
5. The legend identifying individual terms on the plot.
6. A box that shows elements excluded from the plot. This box is empty in this display because the diagram displays all three elements of the transfer function.
7. A "Legend" box that shows elements displayed in the plot.

7. A "Legend" box that shows elements displayed in the plot.
8. Several check-boxes that allow the user to display how the plots are displayed
9. Also in the upper left is a "Help" tab.

Also shown in the upper right hand corner is the transfer function, H(s).

Modifying what is displayed

The function displayed can be manipulated term by term to illustrate the effect of each term. For example, the zero at the origin can be excluded simply by clicking on it in the lower left hand box. The figure below shows the result.



$$H(s) = \frac{1000}{s + 10}$$

Elements included in plot

Constant = 1e+02 (40 dB) phi=0
Real Pole at -10

Select element to exclude from plot

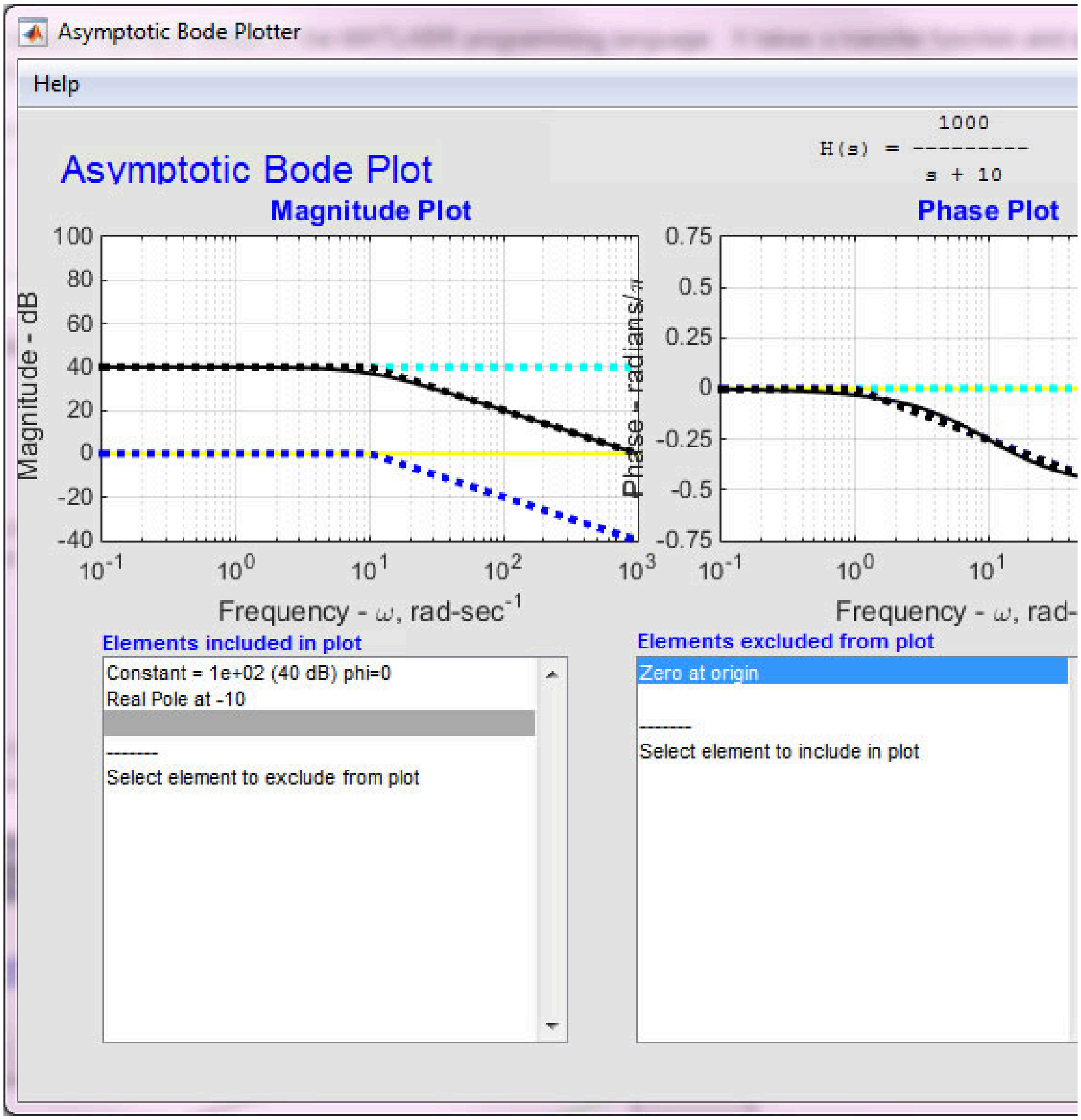
Elements excluded from plot

Zero at origin

Select element to include in plot

Note that the zero at the origin is no longer included in the plot. Each term can be likewise included or excluded by simply clicking on it.

The next plot shows the plot modified to have thicker lines, a grid, phase in radians and with the asymptotic plot of the complete transfer function. In the previous graph, the phase of the asymptotic plot obscured that of the real pole; this is an example when it might be convenient not to show the asymptotic approximation.



Constant = 1e+02 (40 dB) phi=0
Real Pole at -10

Select element to exclude from plot

Zero at origin

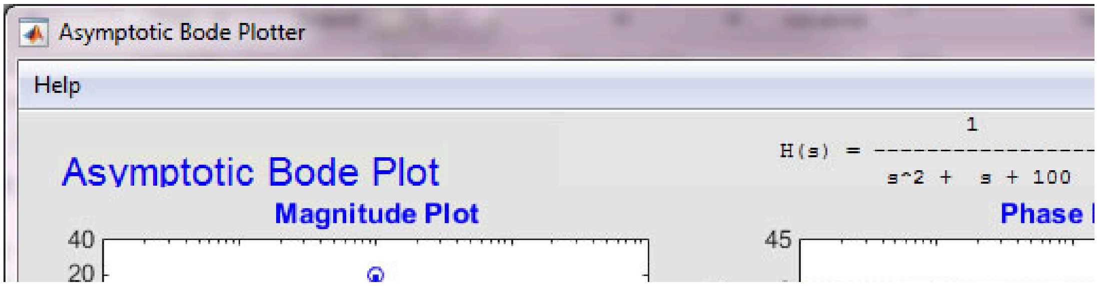
Select element to include in plot

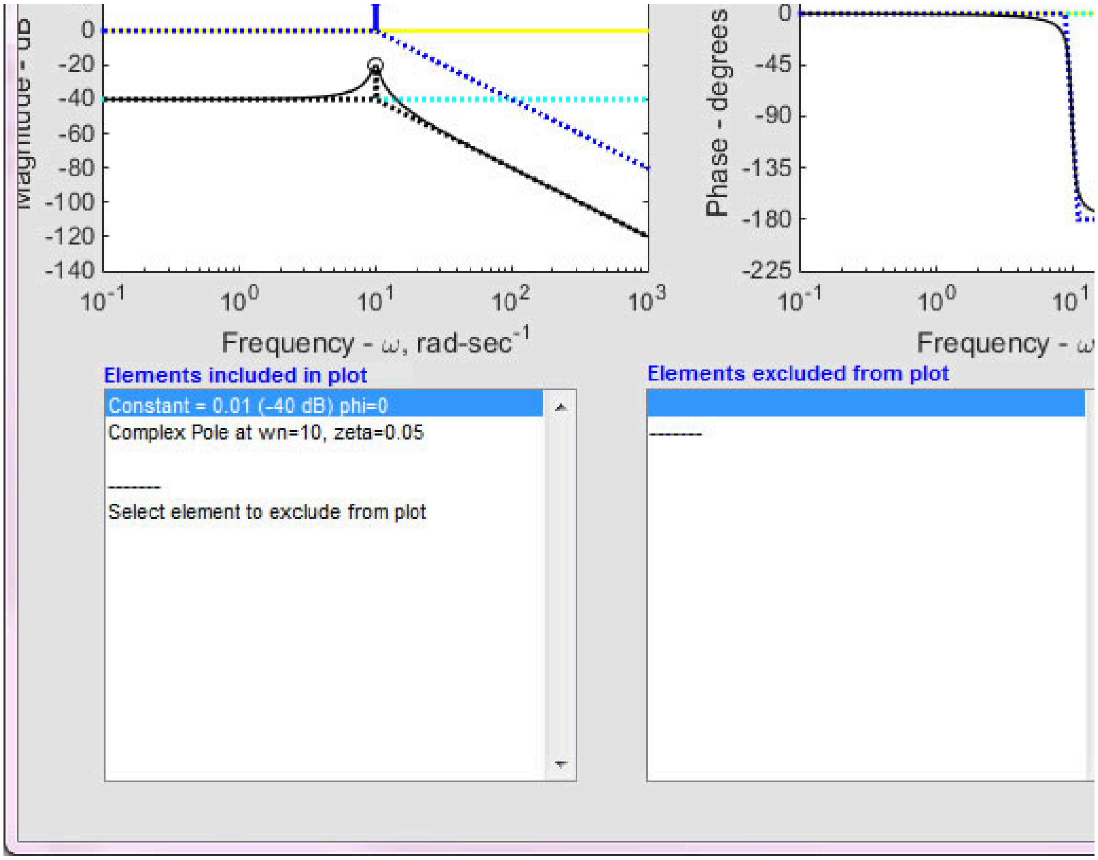
Underdamped terms

Underdamped poles (and zeros) present a difficulty because they cause a peak (dip) in the magnitude plot. The program show this with a simple circle showing the peak height. For example the transfer function

$$H(s) = \frac{1}{s^2 + s + 100}$$

yields the output shown below. The peak due to the underdamped pole is clearly shown.





Elements included in plot

Constant = 0.01 (-40 dB) phi=0
Complex Pole at wn=10, zeta=0.05

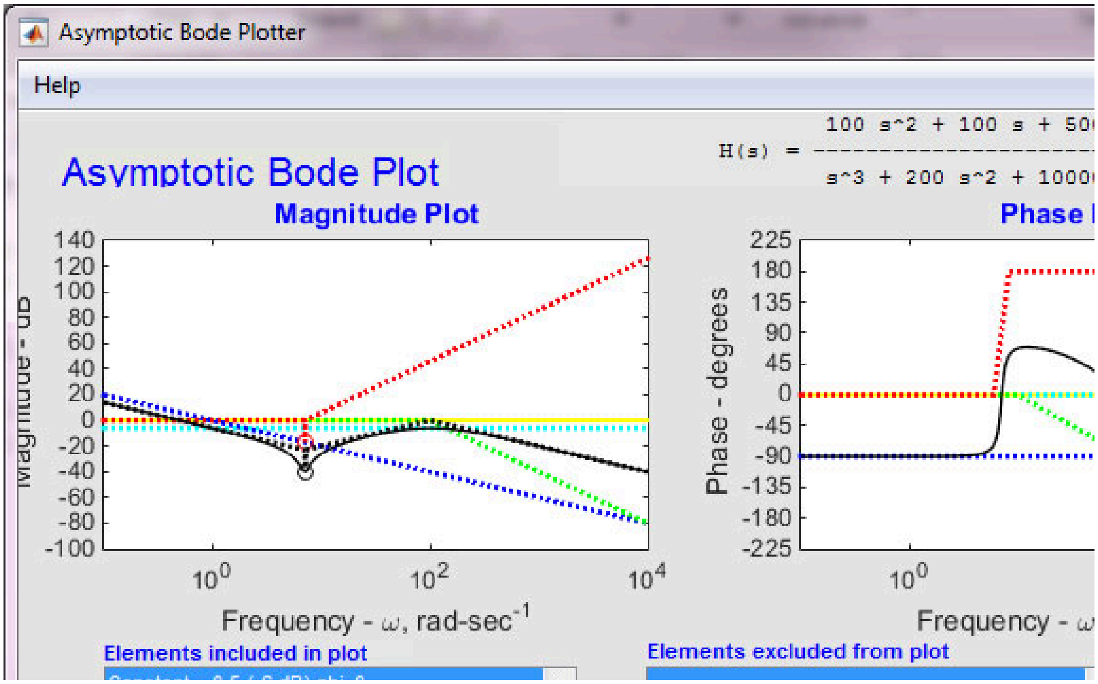
Select element to exclude from plot

Elements excluded from plot

A more complicated example

The example below is more complicated. It shows underdamped terms, repeated poles, and a pole at the origin.

$$H(s) = 100\frac{s^2+s+50}{s^3+200s^2+10000s} = 100\frac{s^2+s+50}{s(s+100)^2}$$



```
        100 s^2 + 100 s + 50
H(s) = ---------------------
        s^3 + 200 s^2 + 1000
```

Elements included in plot

Elements excluded from plot

Make your own Bode plot paper

The code for BodePaper.m is available at https://github.com/echeever/BodePlotGui

When making Bode plots one needs two pieces of semi-logarithmic paper, one for the magnitude plot and one for the phase. The program described here, BodePaper.m, can be used to make paper. Download it and save it so that MatLab can find it (from the Matlab menu you can go to *File→Set Path* and include the directory where you stored the BodePaper.m file.) . There is also a fine in the repository called BodeMagPaper.m that creates only a magnitude plot.

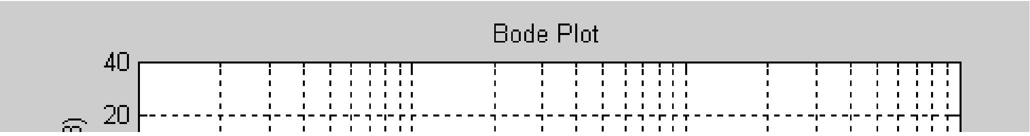
The syntax for calling is given by the function's help file.

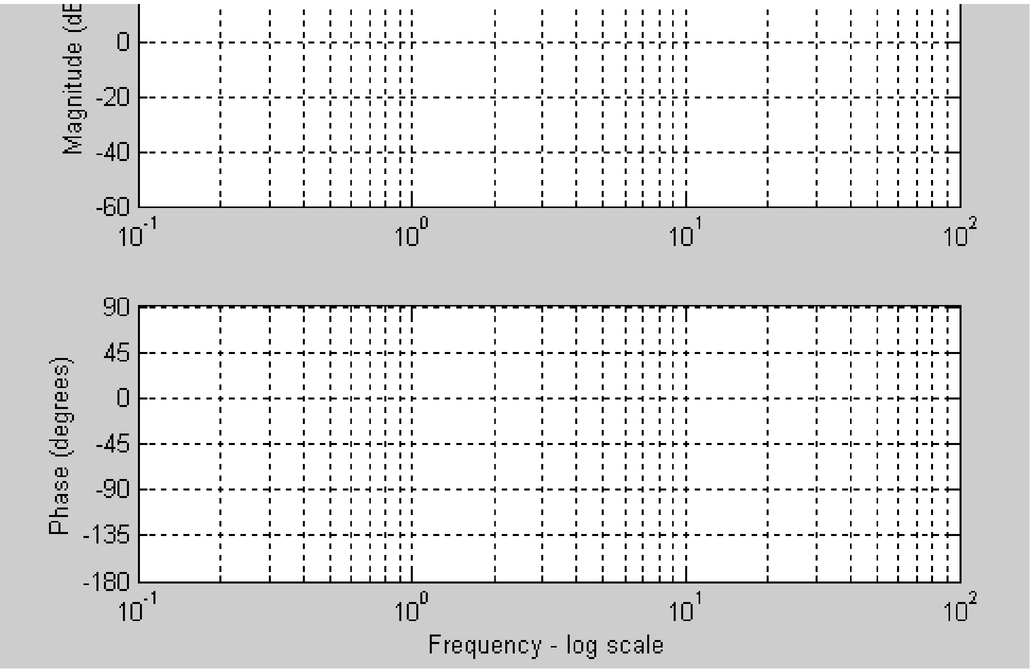
```
>> help BodePaper

BodePaper is Matlab code to generate graph paper fo
two semilog graphs for making Bode plots. The top p
units on the vertical axis is set to dB. The bottom
units on the phase plot can be radians or degrees,
user. The default is degrees.

The correct calling syntax is:
BodePaper(om_lo, om_hi, dB_lo, dB_hi, ph_lo, ph_hi,
    om_lo the low end of the frequency scale. This
        rad/sec or Hz. No units are displayed on th
    om_hi the high end of the frequency scale.
    dB_lo the bottom end of the dB scale.
    dB_hi the top end of the dB scale.
    ph_lo the bottom end of the phase scale.
    ph_hi the top end of the phase scale.
    UseRad an optional argument. If this argument i
        on the phase plot are radians. If this argu
        or set to zero, the units are degrees.
```

To make paper that goes from 0.1 Hz to 100 Hz, with the magnitude scale going from -60 to 40 dB and the phase from -180 to 90 degrees, the function call would be

```
>> BodePaper(0.1,100,-60,40,-180,90)
```

and the paper looks like:

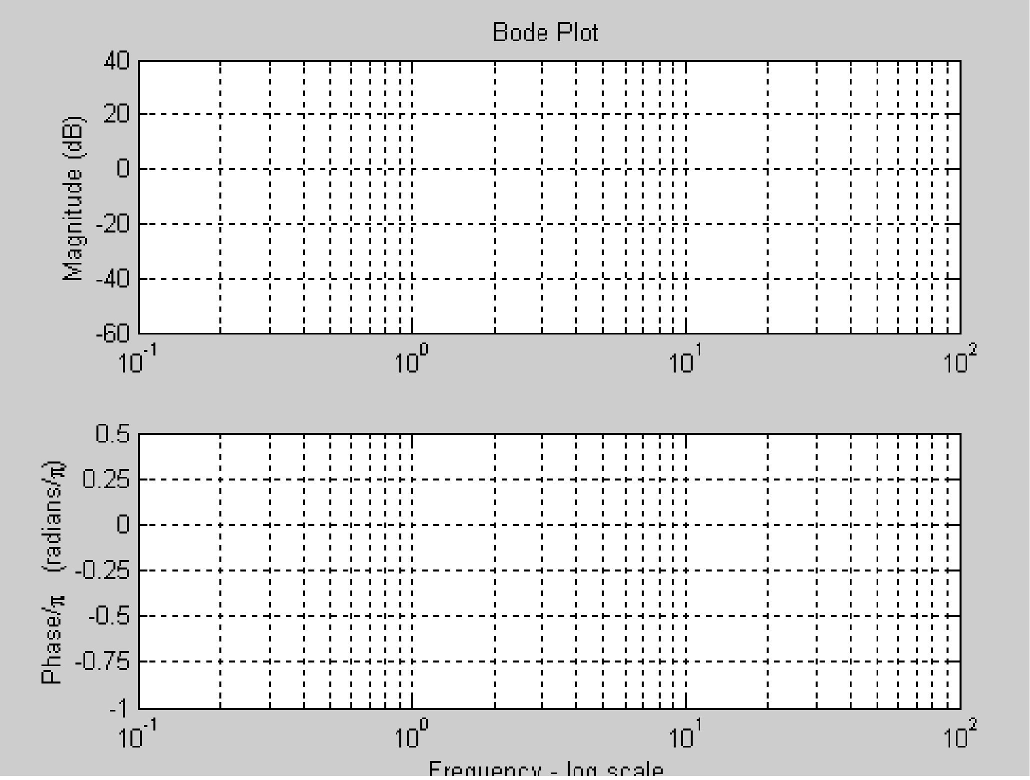




To change the units on phase the function call would be:

```
BodePaper(0.1,100,-60,40,-pi,pi/2,1)
```

and the paper now looks like this:



Referei

Erik Cheever Department of Engineering Swarthmore College

Refere

What Bode Plots Represent: The Frequency Domain

Overview | Freq Domain | Asymptotic plots | Making Plot | Examples | Drawing Tool | BodePlotGui | Rules Table | Printable

Contents

Why Sine Waves?

One of the most commonly used test functions for a circuit or system is the sine (or cosine) wave. This is not because sine waves are a particularly common signal. They are in fact quite rare - the transmission of electricity (a 60 Hz sine wave in the U.S., 50 Hz in much of the rest of the world) is one example. The reason sine waves are important is complex and involve a branch of Mathematics called Fourier Theory. Briefly put: any signal going into a circuit can be represented by a sum of sinusoidal waves of varying frequency and amplitude (often an infinite sum).

This is why sine waves are important. Not because they are common, but because we can represent arbitrarily complex functions using only these very simple function.

Determining system output given input and transfer function

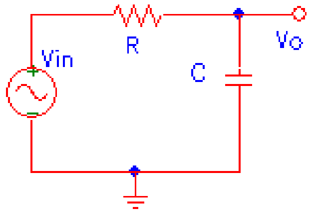
Given that sinusoidal waves are important, how can we analyze the response of a circuit or system to sinusoidal inputs (after all transients have died out - the so-called sinusoidal steady state)? There are many ways to do this, depending on your mathematical sophistication. Let's use a fairly basic explanation that uses phasors. If you are unfamiliar with phasors, a brief introduction is here. A technique using Laplace Transforms is given here.

For a system of the type we are studying (linear constant coefficient) if the input to a system is sinusoidal at a particular frequency, then the output of the system is also a sinusoid at the same frequency, but typically with a different amplitude or phase. Put another way, if the input to a system (described by the transfer function H(s)) is A·cos(ω·t+ϕ) then the output is M·A·cos(ω·t+ϕ+θ). This is likewise true for sine, since it simply a cosine with ϕ=-π/2 radians (or -90°). This is shown below.

simply a cosine with φ=-π/2 radians (or -90°). This is shown below.

In this diagram the magnitude of the sinusoid has changed by a factor of M (which we will take to be a positive real number) and the phase has changed by a factor of θ (a real number, not necessarily positive). It is our task to find the value of M and θ for a particular system, H(s), at a particular frequency, ω. We call M the magnitude of the system (or transfer function) at ω, and we call θ the phase of the system at that frequency.

Using complex impedances it is possible to find the transfer function of a circuit. For example, the circuit below is described by the transfer function, H(s), where s= *jω*.

Circuit	Transfer Function
	$H(s)=\dfrac{V_{out}(s)}{V_{in}(s)}=\dfrac{1}{1+sRC}$

Consider the case where R=2MΩ and C=1µF. In that case:

$$H(s)=\frac{1}{1+2s} \qquad H(j\omega)=\frac{1}{1+j2\omega}$$

Generally we know the input V_{in} and want to find the output V_{out}. We can do this by simple multiplication

$$V_{out}(j\omega)=V_{in}(j\omega)\cdot H(j\omega)=V_{in}(j\omega)\cdot\frac{1}{1+j2\omega}$$

If we have a phasor representation for the input and the transfer function, the multiplication is simple (multiply magnitudes and add phases). Finding the output becomes easy. Try it out.

Interactive Demo

Choose a transfer function.

●

$H(s)=\frac{1}{1+2s}$ $H(j\omega)=\frac{1}{1+j2\omega}$

○

$H(s)=\frac{1.6}{s^2+0.5s+1.6}$ $H(j\omega)=\frac{1.6}{(1.6-\omega^2)+j(0.5\cdot\omega)}$

Set input parameters,
$V_{in}(t)$=A·cos(ω·t+ϕ).

Set ω: 1.000 ω

0 3

Set A: 1 A

0.2 2

Set ϕ: 0 ϕ

-180
180

At ω =1, H(*j*ω) = 1/(1.00 + j2.00) = 0.45∠-63.4° = M∠θ.
Since the input can be represented as 1∠0°,
The output is M·A∠(θ+ϕ) = 0.45∠-63.4°.

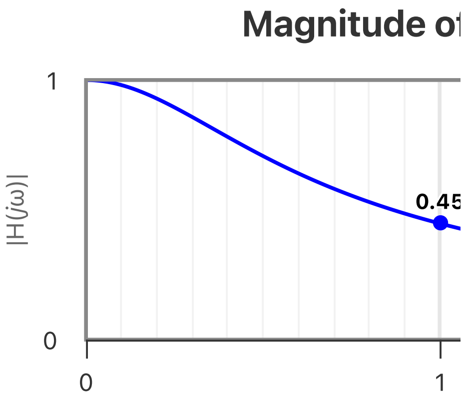
	Magnitude	Phase	Time Domain
H(*j*ω)	0.45	-63.4°	0.45·cos(1·t + -63.4°)
Input	1	0°	1·cos(1·t + 0°)
Output	0.45	-63.4°	0.45·cos(1·t + -63.4°)

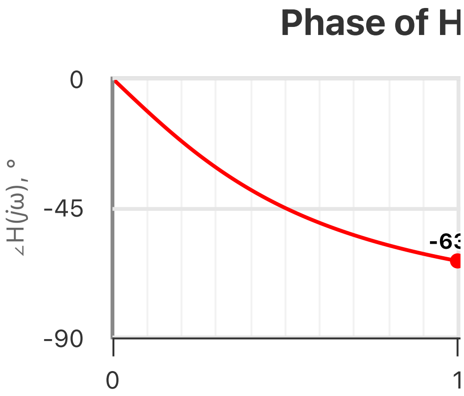
Directions for Use

Use the radio buttons to choose a transfer function, and the sliders to choose the frequency, amplitude and phase of the input (you can also set frequency by clicking and dragging in either of the top two graphs.)

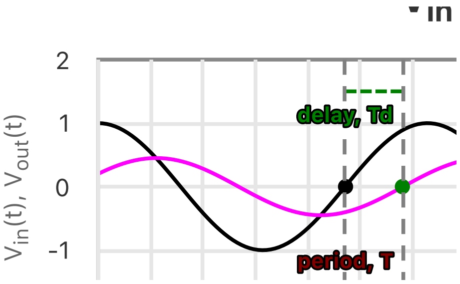
The paragraph below the sliders goes through the calculation of the numerical value of the transfer function at the chosen frequency, and gives H(*j*ω) in terms of magnitude and phase. Note that these are also shown on the top two graphs by a dot. To find the magnitude of the output, simply multiply the magnitude of the input (A) by the magnitude of the transfer function (M). The phase of the output is sum of the input phase (ϕ) and the phase of the transfer function (θ).

The bottom graph shows input, $V_{in}(t)$ in black, and $V_{out}(t)$ in magenta. The period, T (maroon), is shown from one upward zero-crossing of the input function to the next (shown by black dots). The delay T_d (green), is shown from an upward zero crossing of the input to the next upward zero crossing of the output (green dot). The phase is negative (since output lags input) and equal to $-T_d/T·360°$. So if the delay was $T_d=T/4$ (i.e., one quarter of a period) the phase shift would be -90°)





V


In
2
1
0
-1
$V_{in}(t)$, $V_{out}(t)$
delay, Td
period, T

The Asymptotic Bode Diagram: Derivation of Approximations

Overview Freq Domain Asymptotic plots Making Plot Examples Drawing Tool BodePlotGui Rules Table Printable

Skip ahead to interactive demos.

Contents

Skip ahead to interactive demos.

Introduction

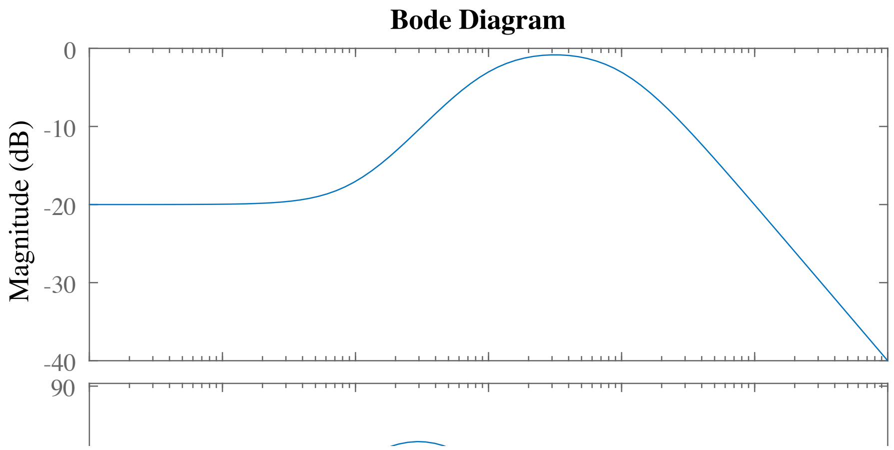
Given an arbitrary transfer function, such as

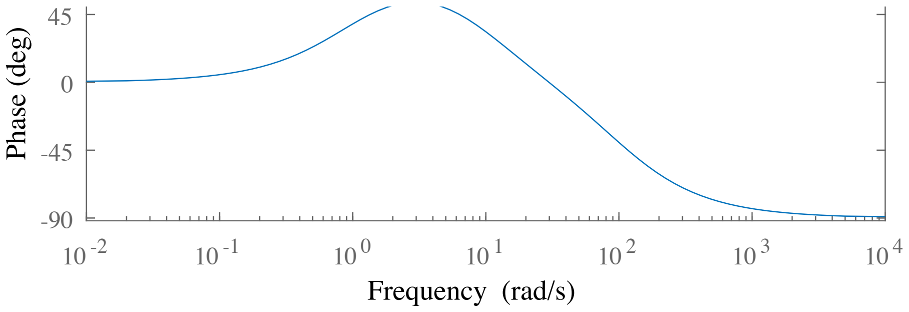
$$H(s) = \frac{100s + 100}{s^2 + 110s + 1000}$$

if you wanted to make a mode plot you could calculate the value of H(s) over a range of frequencies (recall s=j·ω for a Bode plot), and plot them. This is what a computer would naturally do. For example if you use MATLAB® and enter the commands

```
>> mySys=tf(100*[1 1],[1 110 1000])
mySys =
     100 s + 100
  ------------------
  s^2 + 110 s + 1000
>> bode(mySys)
```

you get a plot like the one shown below. The asymptotic solution is given elsewhere.





However, there are reasons to develop a method for sketching Bode diagrams manually. By drawing the plots by hand you develop an understanding about how the locations of poles and zeros effect the shape of the plots. With this knowledge you can predict how a system behaves in the frequency domain by simply examining its transfer function. On the other hand, if you know the shape of transfer function that you want, you can use your knowledge of Bode diagrams to generate the transfer function.

The first task when drawing a Bode diagram by hand is to rewrite the transfer function so that all the poles and zeros are written in the form (1+s/ω_0). The reasons for this will become apparent when deriving the rules for a real pole. A derivation will be done using the transfer function from above, but it is also possible to do a more generic derivation. Let's rewrite the transfer function from above.

$$\begin{aligned}H(s) &= 100\frac{s+1}{(s+10)(s+100)} = 100\frac{1+s/1}{10\cdot(1+s/10)\cdot 100\cdot(1+s/100)}\\ &= 0.1\frac{1+s/1}{(1+s/10)(1+s/100)}\end{aligned}$$

Now let's examine how we can easily draw the magnitude and phase of this function when s=*jω*.

First note that this expression is made up of four terms, a constant (0.1), a zero (at s=-1), and two poles (at s=-10 and s=-100). We can rewrite the function (with s=*jω*) as four individual phasors (i.e., magnitude and phase), each phasor is within a set of square brackets to make them more easily distinguished from each other..

$$\begin{aligned}H(j\omega) &= 0.1\frac{1+j\omega/1}{(1+j\omega/10)(1+j\omega/100)}\\ &= [|0.1|\angle(0.1)]\,\frac{[|1+j\omega/1|\angle(1+j\omega/1)]}{[|1+j\omega/10|\angle(1+j\omega/10)]\,[|1+j\omega/100|\angle(1+j\omega/100)]}\end{aligned}$$

We will show (below) that drawing the magnitude and phase of each individual phasor is fairly straightforward. The difficulty lies in trying to draw the magnitude and phase of the more complicated function, H(*jω*). To start, we will write H(*jω*) as a single phasor:

$$H(j\omega) = \Bigg(|0.1| \qquad \frac{|1+j\omega/1|}{} \Bigg)\left(\angle(0.1)+\angle(1+j\omega/1)-\angle(1+j\omega/1\right.$$

$$H(j\omega) = \left(|0.1| \frac{}{|1+j\omega/10|\,|1+j\omega/100|}\right)\left(\angle(0.1) + \angle(1+j\omega/1) - \angle(1+j\omega/\right.$$

$$= |H(j\omega)|\,\angle H(j\omega)$$

$$|H(j\omega)| = |0.1| \frac{|1+j\omega/1|}{|1+j\omega/10|\,|1+j\omega/100|}$$

$$\angle H(j\omega) = \angle(0.1) + \angle(1+j\omega/1) - \angle(1+j\omega/10) - \angle(1+j\omega/100)$$

Drawing the phase is fairly simple. We can draw each phase term separately, and then simply add (or subtract) them. The magnitude term is not so straightforward because the magnitude terms are *multiplied*, it would be much easier if they were added - then we could draw each term on a graph and just *add* them. We can accomplish this by usinga logarthmic scale (so multiplication and division become addition and subtraction). Instead of a simple logarithm, we will use a deciBel (or dB) scale.

A Magnitude Plot

One way to transform multiplication into addition is by using the logarithm. Instead of using a simple logarithm, we will use a deciBel (named for Alexander Graham Bell). *(Note: Why the deciBel)* The relationship between a quantity, Q, and its deciBel representation, X, is given by:

$$X = 20 \cdot log_{10}(Q)$$

So if Q=100 then X=40; Q=0.01 gives X=-40; X=3 gives Q=1.41; and so on.

If we represent the magnitude of H(s) in deciBels several things happen.

$$|H(s)| = |0.1| \frac{|1+j\omega/1|}{|1+j\omega/10|\,|1+j\omega/100|}$$

$$20 \cdot \log_{10}(|H(s)|) = 20 \cdot \log_{10}\left(|0.1| \frac{|1+j\omega/1|}{|1+j\omega/10|\,|1+j\omega/100|}\right)$$

$$= 20 \cdot \log_{10}(|0.1|) + 20 \cdot \log_{10}(|1+j\omega/1|) + 20 \cdot \log_{10}\left(\frac{}{|1+}\right.$$

$$= 20 \cdot \log_{10}(|0.1|) + 20 \cdot \log_{10}(|1+j\omega/1|) - 20 \cdot \log_{10}(|1+j$$

The advantages of using deciBels (and of writing poles and zeros in the form (1+s/ω_0)) are now revealed. The fact that the deciBel is a logarithmic term transforms the multiplications and divisions of the individual terms to additions and subtsractions. Another benefit is apparent in the last line that reveals just two types of terms, a constant term and terms of the form 20·log10(l1+jω/ω_0l). Plotting the constant term is trivial, however the other terms are not so straightforward. These plots will be discussed below. However, once these plots are drawn for the individual terms, they can simply be added together to get a plot for H(s).

A Phase Plot

If we look at the phase of the transfer function, we see much the same thing: The phase plot is easy to draw if we take our lead from the magnitude plot. First note that the transfer function is made up of four terms. If we want

$$\angle H(s) = \angle(0.1) + \angle(1 + j\omega/1) - \angle(1 + j\omega/10) - \angle(1 + j\omega/100)$$

Again there are just two types of terms, a constant term and terms of the form (1+jω/ω$_0$). Plotting the constant term is trivial; the other terms are discussed below.

A more generic derivation

The discussion above dealt with only a single transfer function. Another derivation that is more general, but a little more complicated mathematically is here.

Making a Bode Diagram

Following the discussion above, the way to make a Bode Diagram is to split the function up into its constituent parts, plot the magnitude and phase of each part, and then add them up. The following gives a derivation of the plots for each type of constituent part. Examples, including rules for making the plots follow in the next document, which is more of a "How to" description of Bode diagrams.

A Constant Term

Consider a constant term:$H(s) = H(j\omega) = K$

Magnitude

Clearly the magnitude is constant as ω varies. $|H(j\omega)| = |K|$

Phase

The phase is also constant. If K is positive, the phase is 0° (or any even multiple of 180°, i.e., ±360°). If K is negative the phase is -180°, or any odd multiple of 180°. We will use -180° because that is what MATLAB® uses. Expressed in radians we can say that if K is positive the phase is 0 radians, if K is negative the phase is -π radians.

Example: Bode Plot of Gain Term

$$H(s) = H(j\omega) = 15$$

$$|H(j\omega)| = |15| = 15 = 23.5\,dB$$

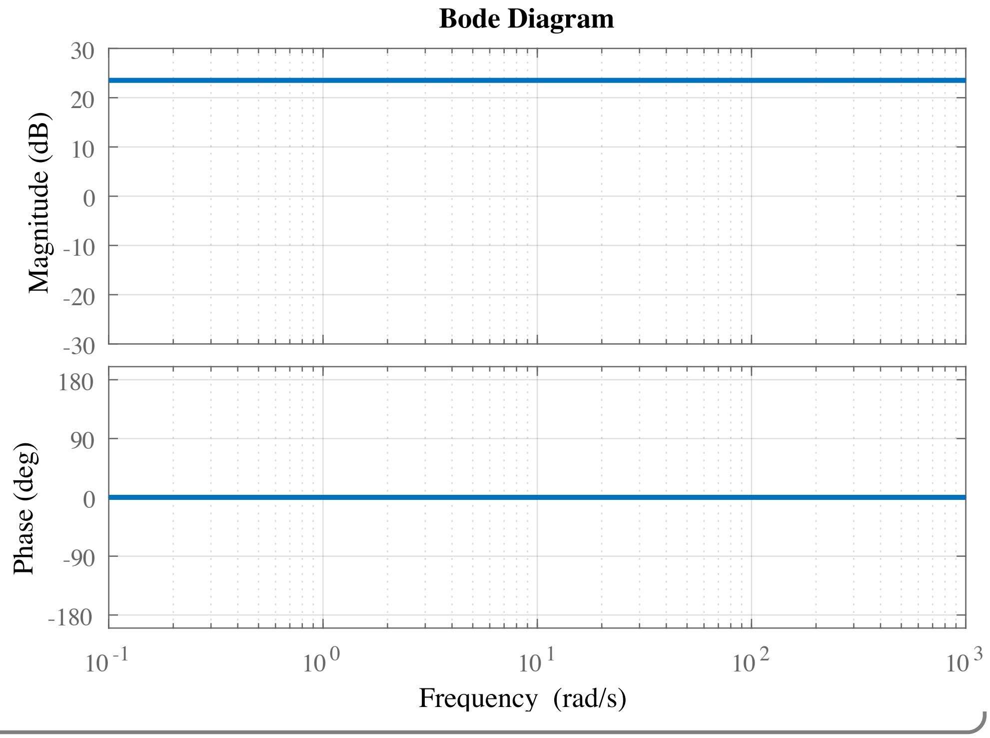
$$\angle H(j\omega) = \angle 15 = 0^\circ$$

$$\angle H(j\omega) = \angle 15 = 0$$

The magnitude (in dB is calculated as

$$20 \cdot \log_{10}(15) = 23.5$$

.



Key Concept: Bode Plot of Gain Term

- For a constant term, the magnitude plot is a straight line.
- The phase plot is also a straight line, either at 0° (for a positive constant) or ±180° (for a negative constant).

Interactive Demo

A Real Pole

Consider a simple real pole : $H(s) = \frac{1}{1+\frac{s}{\omega_0}}$, $\qquad H(j\omega) = \frac{1}{1+j\frac{\omega}{\omega_0}}$

The frequency ω_0 is called the break frequency, the corner frequency or the 3 dB frequency (more on this last name later). The analysis given below assumes ω_0 is positive. For negative ω_0 here.

Magnitude

The magnitude is given by

$$|H(j\omega)| = \left|\frac{1}{1+j\frac{\omega}{}}\right| = \frac{1}{\sqrt{}}$$

$$\sqrt{1^2+\left(\frac{\omega}{\omega_0}\right)}$$

$$\left|H\left(j\omega\right)\right|_{dB}=20\cdot\log_{10}\left(\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}\right)$$

Let's consider three cases for the value of the frequency, and determine the magnitude in each case.:

Case 1) $\omega \ll \omega_0$. This is the low frequency case with $\omega/\omega_0 \rightarrow 0$. We can write an approximation for the magnitude of the transfer function:

$$\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}\approx 1,\text{ and }\left|H\left(j\omega\right)\right|_{dB}\approx 20\cdot\log_{10}\left(\frac{1}{1}\right)=0$$

This low frequency approximation is shown in blue on the diagram below.

Case 2) $\omega \gg \omega_0$. This is the high frequency case with $\omega/\omega_0 \rightarrow \infty$. We can write an approximation for the magnitude of the transfer function:

$$\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}\approx\sqrt{\left(\frac{\omega}{\omega_0}\right)^2}\approx\frac{\omega}{\omega_0},\text{ so}$$
$$\left|H\left(j\omega\right)\right|_{dB}\approx 20\cdot\log_{10}\left(\frac{\omega_0}{\omega}\right)$$

The high frequency approximation is at shown in green on the diagram below. It is a straight line with a slope of -20 dB/decade going through the break frequency at 0 dB (if $\omega=\omega_0$ the approximation simplifies to 0 dB; $\omega=10\cdot\omega_0$ gives an approximate gain of 0.1, or -20 dB and so on). That is, the approximation goes through 0 dB at $\omega=\omega_0$, and for every factor of 10 increase in frequency, the magnitude drops by 20 dB..

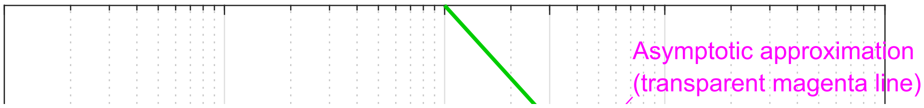
Case 3) $\omega=\omega_0$. At the break frequency

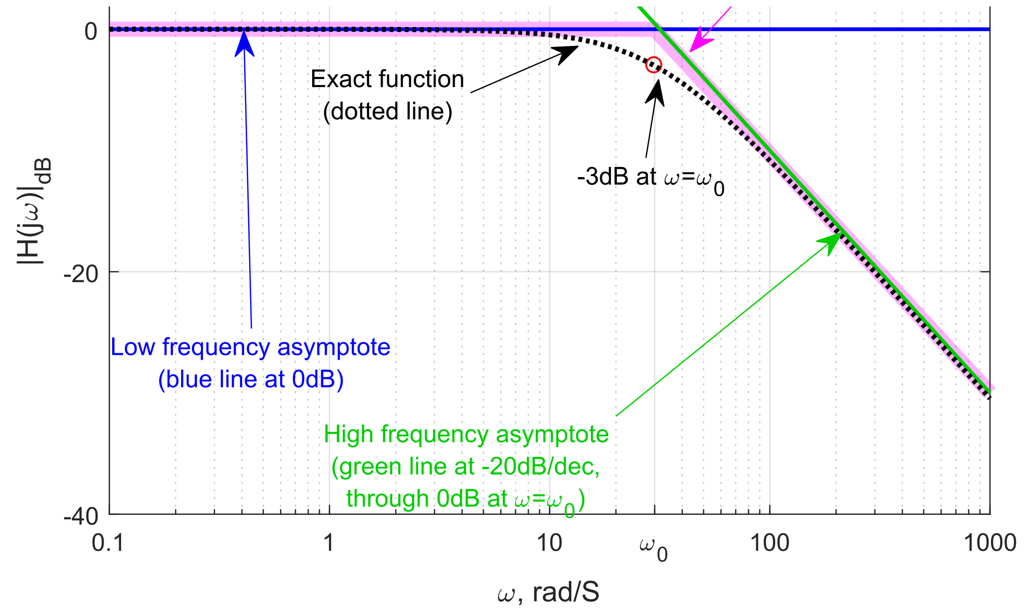
$$\left|H\left(j\omega_0\right)\right|_{dB}=20\cdot\log_{10}\left(\frac{1}{\sqrt{1+\left(\frac{\omega_0}{\omega_0}\right)^2}}\right)=20\cdot\log_{10}\left(\frac{1}{\sqrt{2}}\right)\approx -3\ dB$$

This point is shown as a red circle on the diagram.

To draw a piecewise linear approximation, use the low frequency asymptote up to the break frequency, and the high frequency asymptote thereafter.

Magnitude of Simple Real Pole





The resulting asymptotic approximation is shown highlighted in transparent magenta. The maximum error between the asymptotic approximation and the exact magnitude function occurs at the break frequency and is approximately -3 dB.

Magnitude of a real pole: The piecewise linear asymptotic Bode plot for magnitude is at 0 dB until the break frequency and then drops at 20 dB per decade as frequency increases (i.e., the slope is -20 dB/decade).

Phase

The phase of a single real pole is given by is given by

$$\angle H\left(j\omega\right)=\angle\left(\frac{1}{1+j\frac{\omega}{\omega_0}}\right)=-\angle\left(1+j\frac{\omega}{\omega_0}\right)=-\arctan\left(\frac{\omega}{\omega_0}\right)$$

Let us again consider three cases for the value of the frequency:

Case 1) $\omega<<\omega_0$. This is the low frequency case with $\omega/\omega_0\rightarrow 0$. At these frequencies We can write an approximation for the phase of the transfer function

$$\angle H\left(j\omega\right)\approx-\arctan(0)=0^\circ=0\ rad$$

The low frequency approximation is shown in blue on the diagram below.

Case 2) $\omega>>\omega_0$. This is the high frequency case with $\omega/\omega_0\rightarrow\infty$. We can write an approximation for the phase of the transfer function

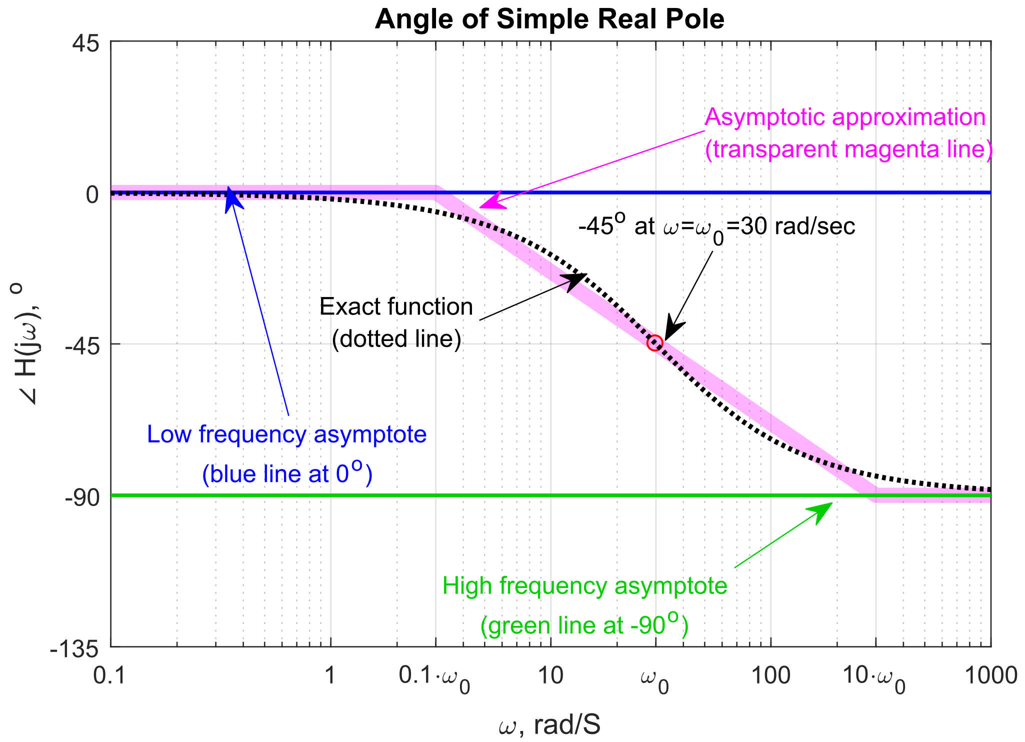
$$\angle H\left(j\omega\right)\approx-\arctan(\infty)=-90^\circ=-\frac{\pi}{2}\ rad$$

The high frequency approximation is at shown in green on the diagram below. It is a horizontal line at -90°.

Case 3) $\omega=\omega_0$. The break frequency. At this frequency

$$\angle H(j\omega) = -\arctan(1) = -45^\circ = -\frac{\pi}{4}\ rad$$

This point is shown as a red circle on the diagram.



A piecewise linear approximation is not as easy in this case because the high and low frequency asymptotes don't intersect. Instead we use a rule that follows the exact function fairly closely, but is also somewhat arbitrary. Its main advantage is that it is easy to remember.

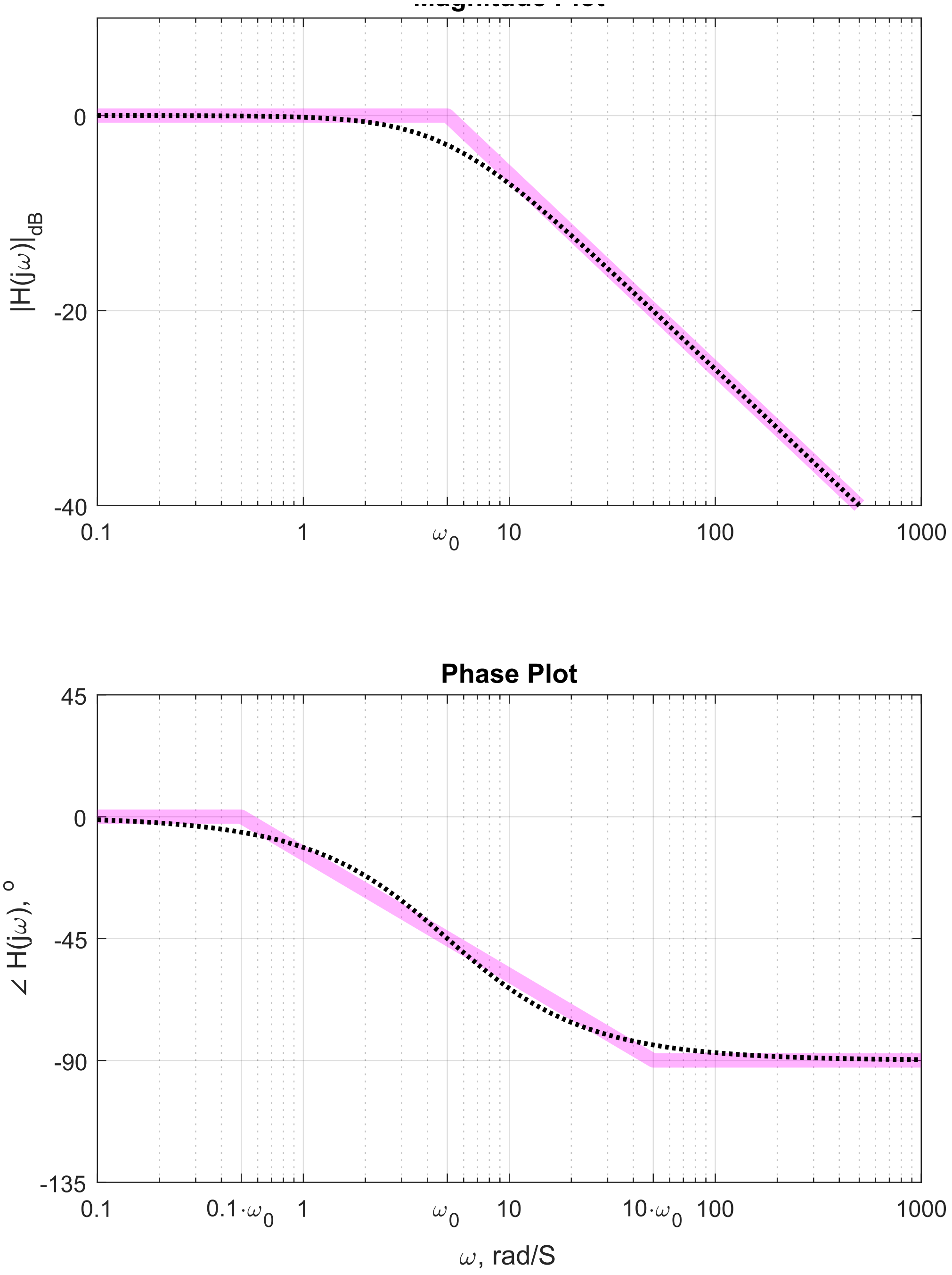
Phase of a real pole: The piecewise linear asymptotic Bode plot for phase follows the low frequency asymptote at 0° until one tenth the break frequency ($0.1\cdot\omega_0$) then decrease linearly to meet the high frequency asymptote at ten times the break frequency ($10\cdot\omega_0$). This line is shown above. Note that there is no error at the break frequency and about 5.7° of error at $0.1\cdot\omega_0$ and $10\cdot\omega_0$ the break frequency.

Example: Real Pole

The first example is a simple pole at 5 radians per second. The asymptotic approximation is magenta, the exact function is a dotted black line.

$$H(s) = \frac{1}{1+\frac{s}{5}}$$

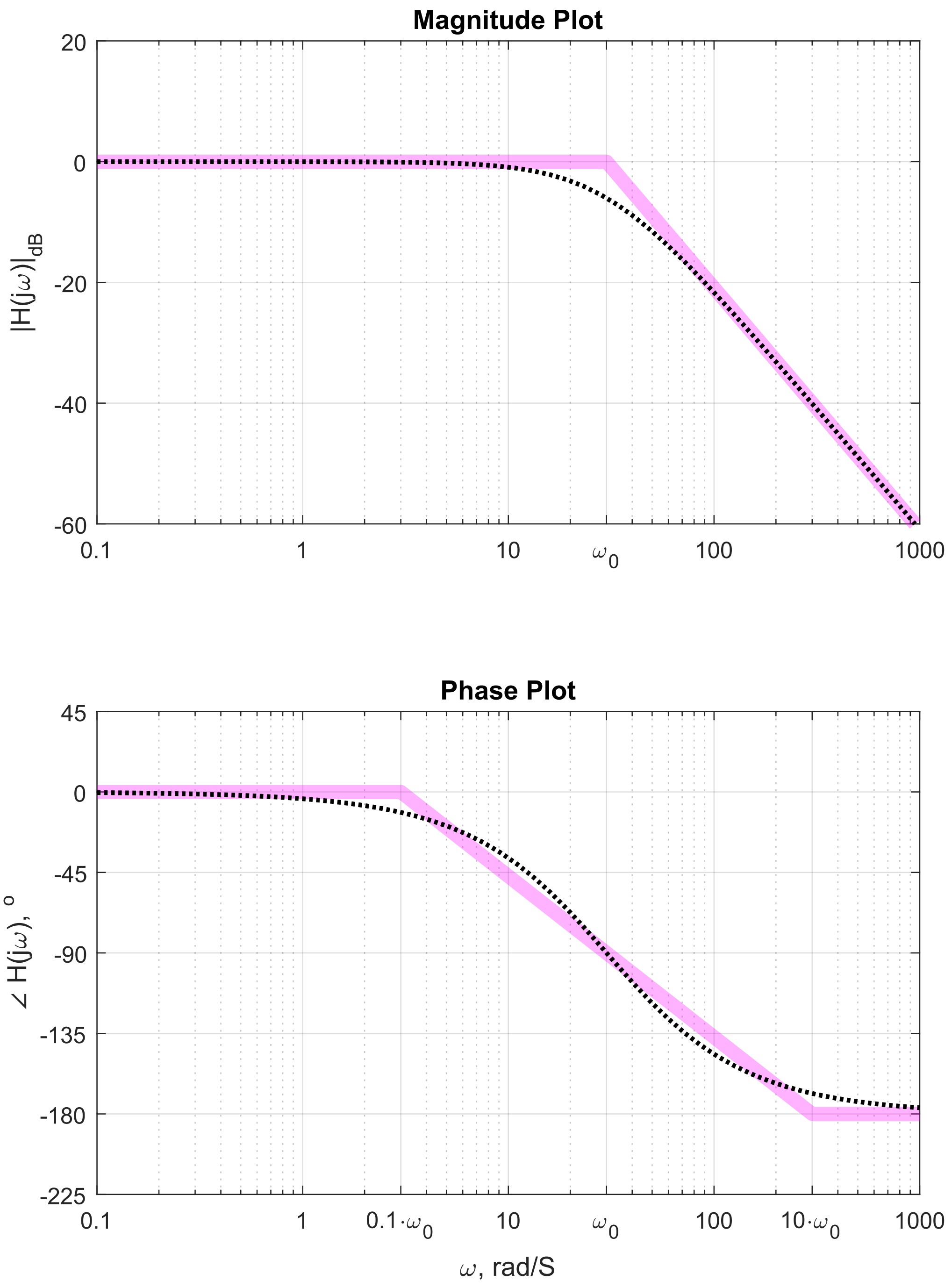
Magnitude Plot



Example: Repeated Real Pole

The second example shows a double pole at 30 radians per second. Note that the slope of the asymptote is -40 dB/decade and the phase goes from 0 to -180°. The effect of repeating a pole is to double the slope of the magnitude to -40 dB/decade and the slope of the phase to -90°/decade.

$$H(s) = \frac{1}{\left(1 + \frac{s}{30}\right)^2}$$



Key Concept: Bode Plot for Real Pole

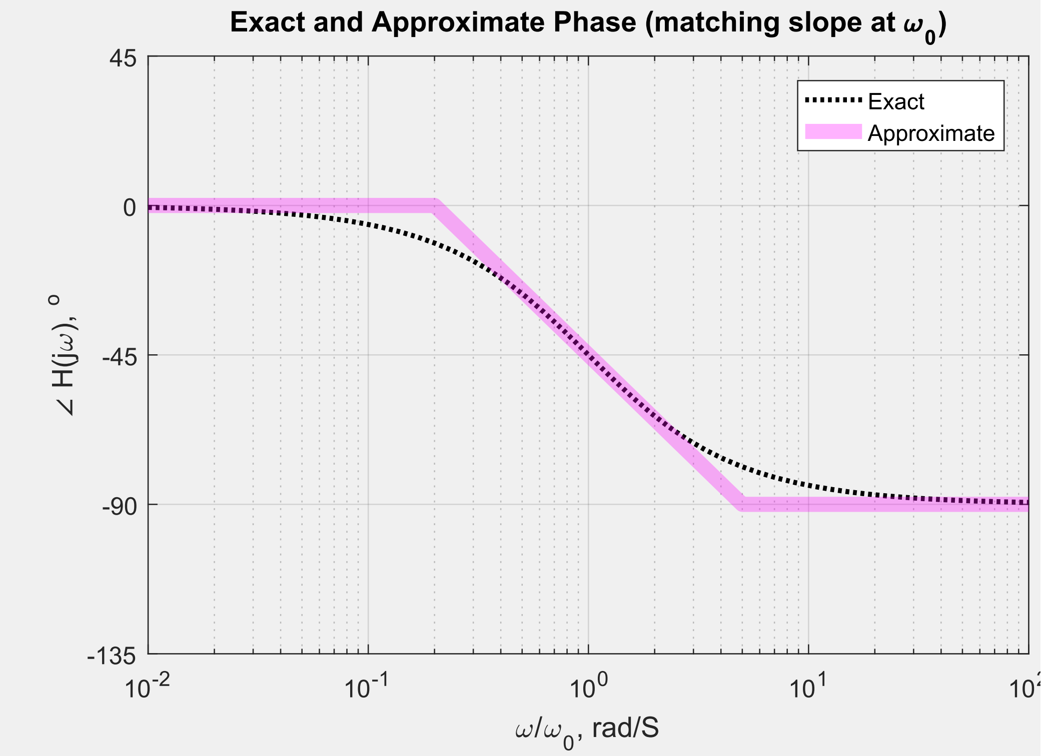
- For a simple real pole the piecewise linear asymptotic Bode plot for magnitude is at 0 dB until the break frequency and then drops at 20 dB per decade (i.e., the slope is -20 dB/decade). An nth order pole has a slope of -20·n dB/decade.
- The phase plot is at 0° until one tenth the break frequency and then drops linearly to -90° at ten times the break frequency. An nth order pole drops to -90°·n.

The analysis given above assumes ω_0 is positive. For negative ω_0 here.

Interactive Demo

Aside: a different formulation of the phase approximation

There is another approximation for phase that is occasionally used. The approximation is developed by matching the slope of the actual phase term to that of the approximation at $\omega=\omega_0$. Using math similar to that given here (for the underdamped case) it can be shown that by drawing a line starting at 0° at $\omega=\omega_0/e^{\pi/2}=\omega_0/4.81$ (or $\omega_0\cdot e^{-\pi/2}$) to -90° at $\omega=\omega_0\cdot 4.81$ we get a line with the same slope as the actual function at $\omega=\omega_0$. The approximation described previously is much more commonly used as is easier to remember as a line drawn from 0° at $\omega_0/5$ to -90° at $\omega_0\cdot 5$, and easier to draw on semi-log paper. The latter is shown on the diagram below.



Although this method is more accurate in the region around $\omega=\omega_0$ there is a larger maximum error (more than 10°) near $\omega_0/5$ and $\omega_0\cdot 5$ when compared to the method described previously.

A Real Zero

The piecewise linear approximation for a zero is much like that for a pole Consider a simple zero: $H(s)=1+\frac{s}{\omega_0}, \quad H(j\omega)=1+j\frac{\omega}{\omega_0}$.

Magnitude

The development of the magnitude plot for a zero follows that for a pole. Refer to the previous section for details. The magnitude of the zero is given by

$$\left|H\left(j\omega\right)\right| = \left|1 + j\frac{\omega}{\omega_0}\right|$$

Again, as with the case of the real pole, there are three cases:

1. At low frequencies, $\omega << \omega_0$, the gain is approximately 1 (or 0 dB).
2. At high frequencies, $\omega >> \omega_0$, the gain increases at 20 dB/decade and goes through the break frequency at 0 dB.
3. At the break frequency, $\omega = \omega_0$, the gain is about 3 dB.

Magnitude of a Real Zero: For a simple real zero the piecewise linear asymptotic Bode plot for magnitude is at 0 dB until the break frequency and then increases at 20 dB per decade (i.e., the slope is +20 dB/decade).

Phase

The phase of a simple zero is given by:

$$\angle H\left(j\omega\right) = \angle\left(1 + j\frac{\omega}{\omega_0}\right) = \arctan\left(\frac{\omega}{\omega_0}\right)$$

The phase of a single real zero also has three cases (which can be derived similarly to those for the real pole, given above):

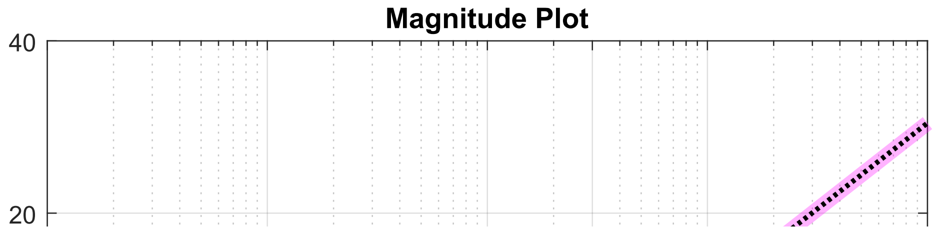
1. At low frequencies, $\omega << \omega_0$, the phase is approximately zero.
2. At high frequencies, $\omega >> \omega_0$, the phase is +90°.
3. At the break frequency, $\omega = \omega_0$, the phase is +45°.

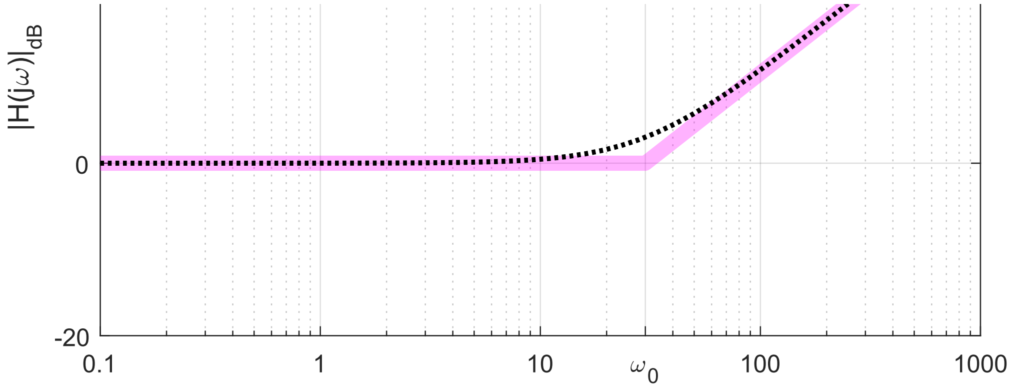
Phase of a Real Zero: Follow the low frequency asymptote at 0° until one tenth the break frequency ($0.1\ \omega_0$) then increase linearly to meet the high frequency asymptote at ten times the break frequency ($10\ \omega_0$).

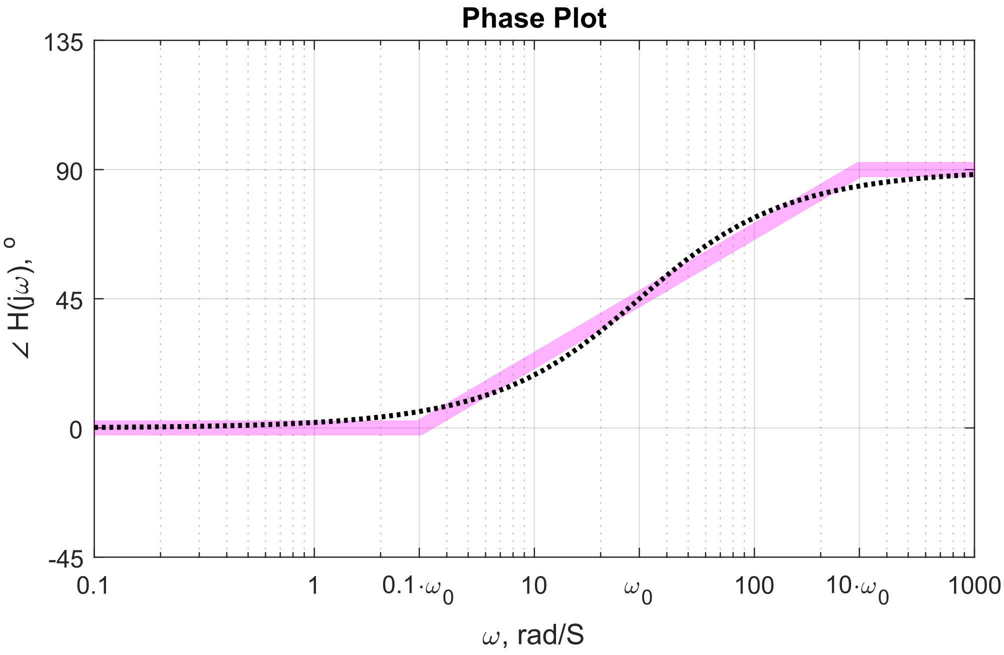
Example: Real Zero

This example shows a simple zero at 30 radians per second. The asymptotic approximation is magenta, the exact function is the dotted black line.

$$H(s) = 1 + \frac{s}{30}$$







Key Concept: Bode Plot of Real Zero:

- The plots for a real zero are like those for the real pole but mirrored about 0dB or 0°.
- For a simple real *zero* the piecewise linear asymptotic Bode plot for magnitude is at 0 dB until the break frequency and then *rises* at +20 dB per decade (i.e., the slope is +20 dB/decade). An n th order *zero* has a slope of +20·n dB/decade.
- The phase plot is at 0° until one tenth the break frequency and then *rises* linearly to +90° at ten times the break frequency. An nth order *zero rises* to +90°·n.

The analysis given above assumes the ω_0 is positive. For negative ω_0 here.

Interactive Demo

A Pole at the Origin

A pole at the origin is easily drawn exactly. Consider

$$H\left(s\right)=\frac{1}{s},\quad H\left(j\omega\right)=\frac{1}{j\omega}=-\frac{j}{\omega}$$

Magnitude

The magnitude is given by

$$\left|H\left(j\omega\right)\right|=\left|-\frac{j}{\omega}\right|=\frac{1}{\omega}$$

$$\left|H\left(j\omega\right)\right|_{dB}=20\cdot\log_{10}\left(\frac{1}{\omega}\right)=-20\cdot\log_{10}\left(\omega\right)$$

In this case there is no need for approximate functions and asymptotes, we can plot the exact funtion. The function is represented by a straight line on a Bode plot with a slope of -20 dB per decade and going through 0 dB at 1 rad/ sec. It also goes through 20 dB at 0.1 rad/sec, -20 dB at 10 rad/sec... Since there are no parameters (i.e., ω_0) associated with this function, it is always drawn in exactly the same manner.

Magnitude of Pole at the Origin: Draw a line with a slope of -20 dB/decade that goes through 0 dB at 1 rad/sec.

Phase

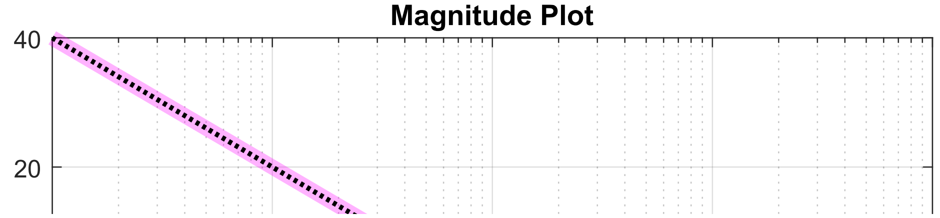
The phase of a simple zero is given by (H(jω) is a negative imaginary number for all values of ω so the phase is always -90°):

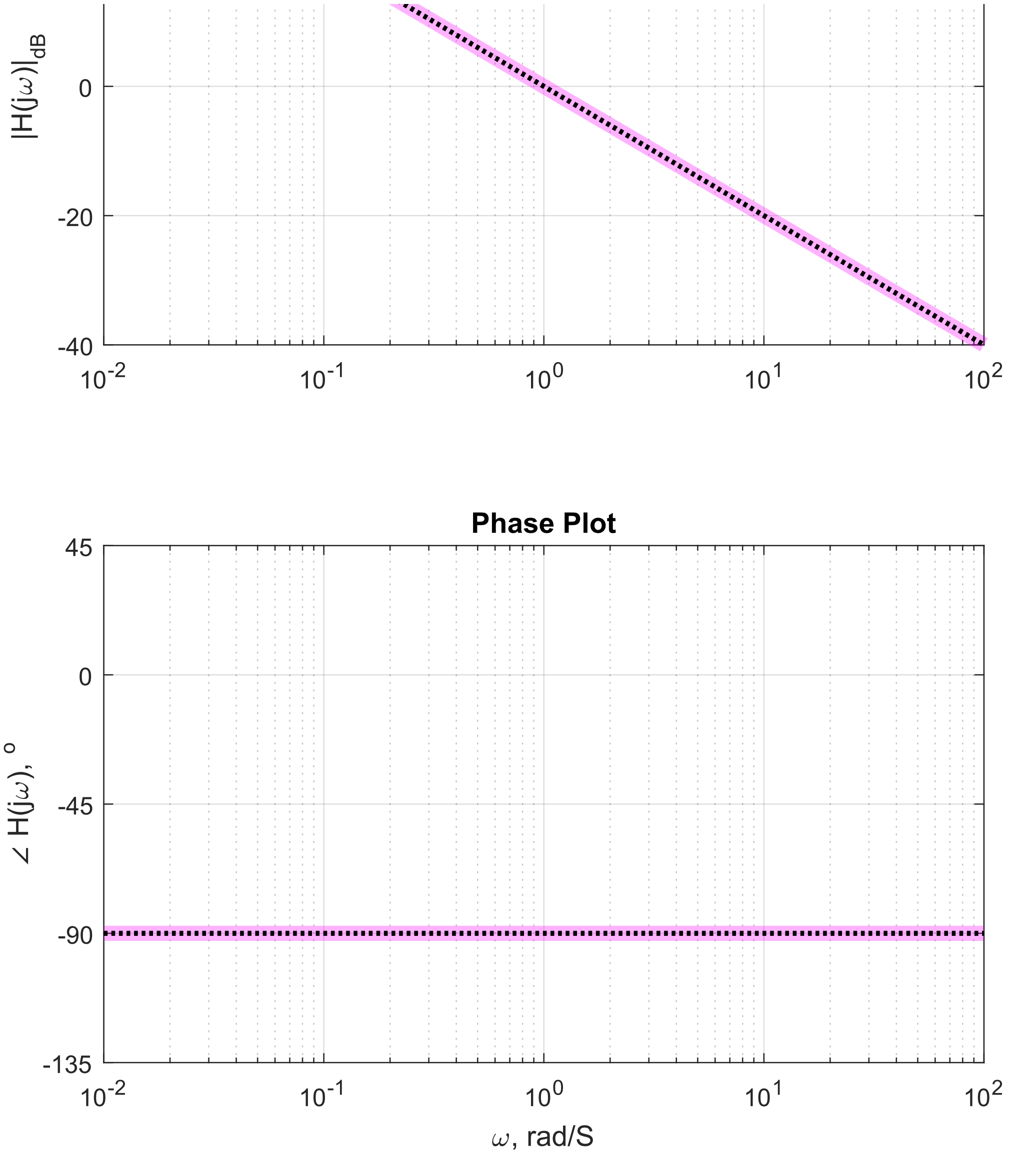
$$\angle H\left(j\omega\right)=\angle\left(-\frac{j}{\omega}\right)=-90^{\circ}$$

Phase of pole at the origin: The phase for a pole at the origin is -90°.

Example: Pole at Origin

This example shows a simple pole at the origin. The exact (dotted black line) is the same as the approximation (magenta).





Key Concept: Bode Plot for Pole at Origin

No interactive demo is provided because the plots are always drawn in the same way.

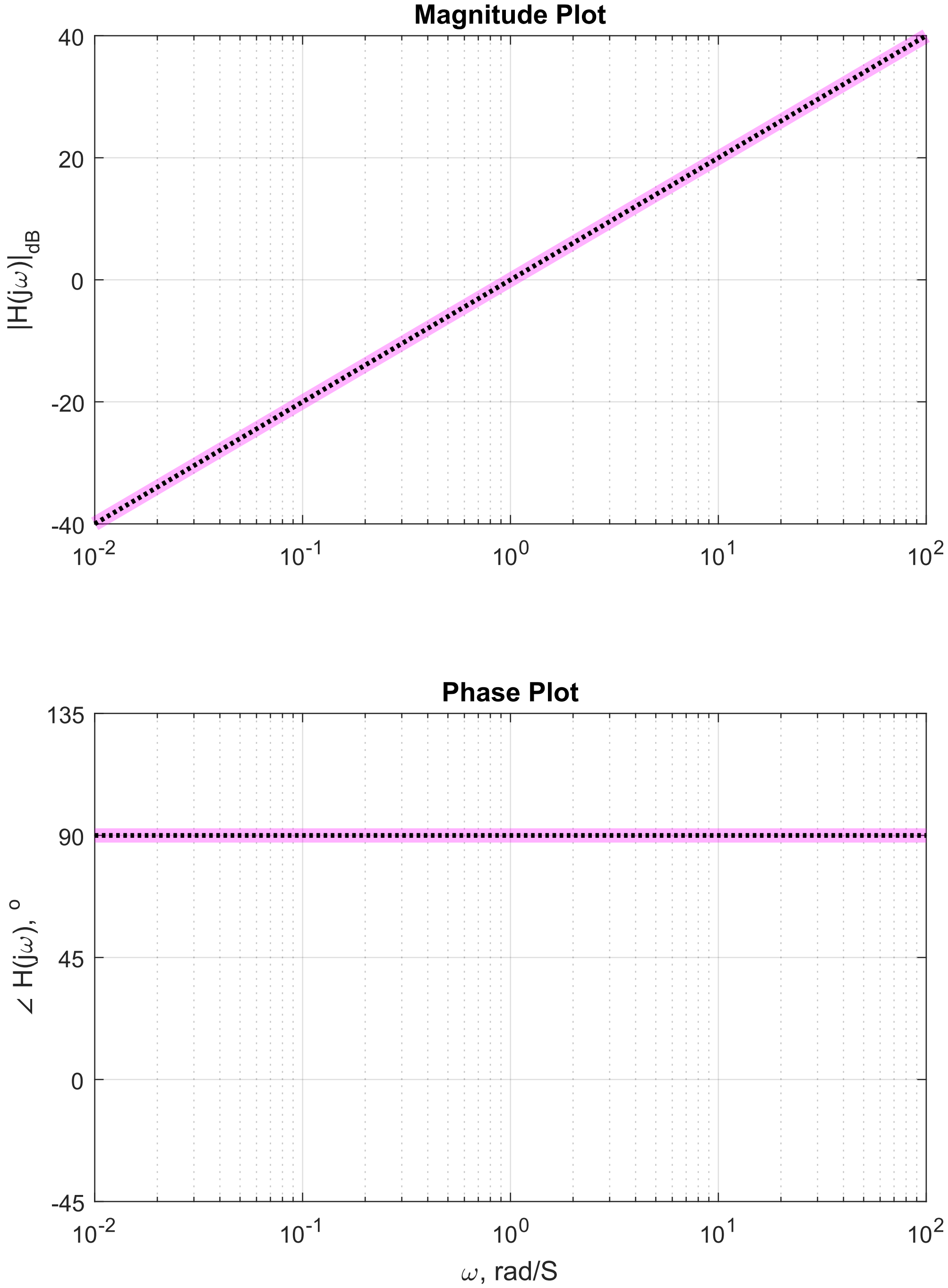
- For a simple pole at the origin draw a straight line with a slope of -20 dB per decade and going through 0 dB at 1 rad/ sec.
- The phase plot is at -90°.
- The magnitude of an n^{th} order pole has a slope of -20·n dB/decade and a constant phase of -90°·n.

A Zero at the Origin

A zero at the origin is just like a pole at the origin but the magnitude increases with increasing ω, and the phase is +90° (i.e. simply mirror the graphs for the pole around the origin around 0dB or 0°).

Example: Zero at Origin

This example shows a simple zero at the origin. The exact (dotted black line) is the same as the approximation (magenta).



Key Concept: Bode Plot for Zero at Origin

- The plots for a zero at the origin are like those for the pole but mirrored about 0dB or 0°.
- For a simple zero at the origin draw a straight line with a slope of +20 dB per decade and going through 0 dB at 1 rad/ sec.
- The phase plot is at +90°.

- The magnitude of an n^th^ order zero has a slope of +20·n dB/decade and a constant phase of +90°·n.

A Complex Conjugate Pair of Poles

The magnitude and phase plots of a complex conjugate (underdamped) pair of poles is more complicated than those for a simple pole. Consider the transfer function (with 0<ζ<1):

$$H(s)=\frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}=\frac{1}{\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1}$$

The analysis given below assumes the ζ is positive. For negative ζ see here.

Magnitude

The magnitude is given by

$$|H(j\omega)|=\left|\frac{1}{\left(\frac{j\omega}{\omega_0}\right)^2+2\zeta\left(\frac{j\omega}{\omega_0}\right)+1}\right|=\left|\frac{1}{-\left(\frac{\omega}{\omega_0}\right)^2+j2\zeta\left(\frac{\omega}{\omega_0}\right)+1}\right|=\left|\frac{}{\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right.}\right.$$

$$=\frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2+\left(2\zeta\frac{\omega}{\omega_0}\right)^2}}$$

$$|H(j\omega)|_{dB}=-20\cdot\log_{10}\left(\sqrt{\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2+\left(2\zeta\frac{\omega}{\omega_0}\right)^2}\right)$$

As before, let's consider three cases for the value of the frequency:

Case 1) ω<<ω_0. This is the low frequency case. We can write an approximation for the magnitude of the transfer function

$$|H(j\omega)|_{dB}=-20\cdot\log_{10}(1)=0$$

The low frequency approximation is shown in red on the diagram below.

Case 2) ω>>ω_0. This is the high frequency case. We can write an approximation for the magnitude of the transfer function

$$\left|H(j\omega)\right|_{dB} = -20\cdot\log_{10}\left(\left(\frac{\omega}{\omega_0}\right)^2\right) = -40\cdot\log_{10}\left(\frac{\omega}{\omega_0}\right)$$

The high frequency approximation is at shown in green on the diagram below. It is a straight line with a slope of -40 dB/decade going through the break frequency at 0 dB. That is, for every factor of 10 increase in frequency, the magnitude drops by 40 dB.

Case 3) $\omega \approx \omega_0$. It can be shown that a peak occurs in the magnitude plot near the break frequency. The derivation of the approximate amplitude and location of the peak are given here. We make the approximation that a peak exists only when

$$0<\zeta<0.5$$

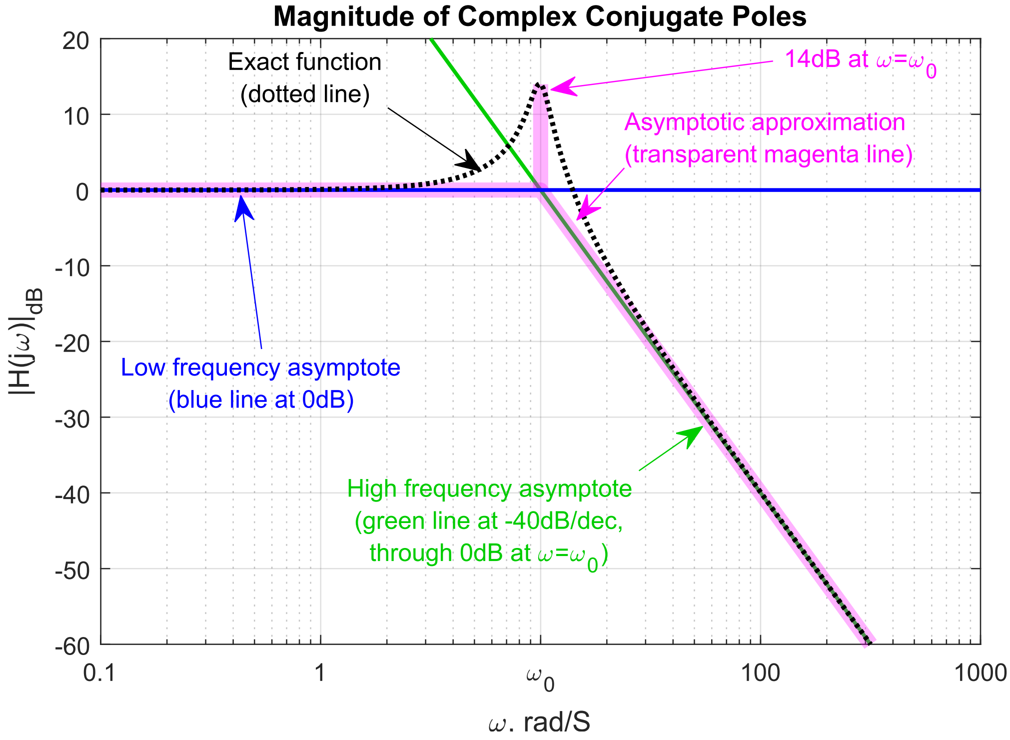
and that the peak occurs at ω_0 with height 1/(2·ζ).

To draw a piecewise linear approximation, use the low frequency asymptote up to the break frequency, and the high frequency asymptote thereafter. If ζ<0.5, then draw a peak of amplitude 1/(2·ζ) Draw a smooth curve between the low and high frequency asymptote that goes through the peak value.

As an example for the curve shown below ω_0=10, ζ=0.1,

$$H(s) = \frac{1}{\frac{s^2}{100}+0.02\zeta s+1} = \frac{1}{\left(\frac{s}{10}\right)^2+0.2\left(\frac{s}{10}\right)+1} = \frac{1}{\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1}$$

The peak will have an amplitude of 1/(2·ζ)=5.00 or 14 dB.



The resulting asymptotic approximation is shown as a black dotted line, the exact response is a black solid line.

Magnitude of Underdamped (Complex) poles: Draw a 0 dB at low frequencies until the break frequency, ω_0, and then drops with a slope of -40 dB/decade. If $\zeta<0.5$ we draw a peak of height at ω_0, otherwise no peak is drawn.

$$|H(j\omega_0)| \approx \frac{1}{2\zeta}, \quad |H(j\omega_0)|_{dB} \approx -20 \cdot \log_{10}(2\zeta)$$

Note: The actual height of the peak and its frequency are both slightly less than the approximations given above. An in depth discussion of the magnitude and phase approximations (along with some alternate approximations) are given here.

Phase

The phase of a complex conjugate pole is given by is given by

$$\angle H(j\omega) = \angle\left(\frac{1}{\left(\frac{j\omega}{\omega_0}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_0}\right) + 1}\right) = -\angle\left(\left(\frac{j\omega}{\omega_0}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_0}\right) + 1\right) = -\angle\ ($$

$$= -\arctan\left(\frac{2\zeta\frac{\omega}{\omega_0}}{1-\left(\frac{\omega}{\omega_0}\right)^2}\right)$$

Let us again consider three cases for the value of the frequency:

Case 1) $\omega \ll \omega_0$. This is the low frequency case. At these frequencies We can write an approximation for the phase of the transfer function

$$\angle H(j\omega) \approx -\arctan\left(\frac{2\zeta\omega}{\omega_0}\right) \approx -\arctan(0) = 0^\circ = 0\ rad$$

The low frequency approximation is shown in red on the diagram below.

Case 2) $\omega \gg \omega_0$. This is the high frequency case. We can write an approximation for the phase of the transfer function

$$\angle H(j\omega) \approx -180^\circ = -\pi\ rad$$

Note: this result makes use of the fact that the arctan function returns a result in quadrant 2 since the imaginary part of H(jω) is negative and the real part is positive.

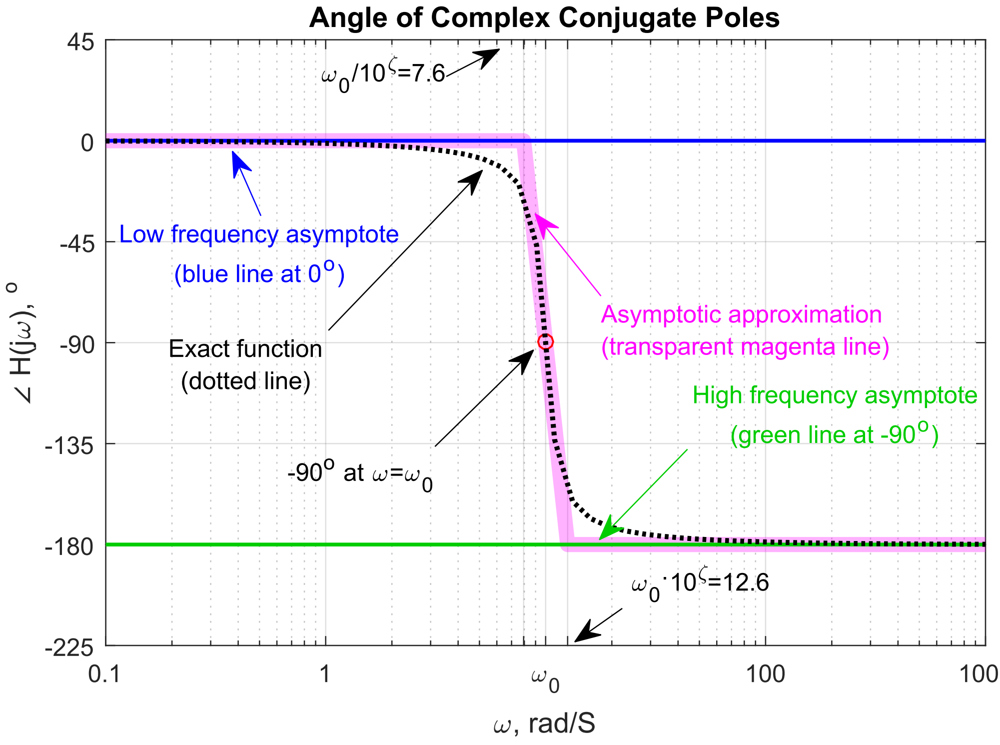
The high frequency approximation is at shown in green on the diagram below. It is a straight line at -180°.

Case 3) $\omega=\omega_0$. The break frequency. At this frequency

$$\angle H(j\omega_0) = -90^\circ$$

The asymptotic approximation is shown below for ω_0=10, ζ=0.1, followed by an explanation

$$H(s) = \frac{1}{\frac{s^2}{100} + 0.02\zeta s + 1} = \frac{1}{\left(\frac{s}{10}\right)^2 + 0.2\left(\frac{s}{10}\right) + 1} = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right) + 1}$$



A piecewise linear approximation is a bit more complicated in this case, and there are no hard and fast rules for drawing it. The most common way is to look up a graph in a textbook with a chart that shows phase plots for many values of ζ. Three asymptotic approximations are given here. We will use the approximation that connects the the low frequency asymptote to the high frequency asymptote starting at

$$\omega = \frac{\omega_0}{10^\zeta} = \omega_0 \cdot 10^{-\zeta}$$

and ending at

$$\omega = \omega_0 \cdot 10^\zeta$$

Since $\zeta=0.2$ in this case this means that the phase starts at 0° and then breaks downward at $\omega=\omega_0/10^\zeta=7.9$ rad/sec. The phase reaches -180° at $\omega=\omega_0\cdot 10^\zeta=12.6$ rad/sec.

As a practical matter If $\zeta<0.02$, the approximation can be simply a vertical line at the break frequency. One advantage of this approximation is that it is very easy to plot on semilog paper. Since the number $10\cdot\omega_0$ moves up by a full decade from ω_0, the number $10^\zeta\cdot\omega_0$ will be a fraction ζ of a decade above ω_0. For the example above the corner frequencies for $\zeta=0.1$ fall near ω_0 one tenth of the way between ω_0 and $\omega_0/10$ (at the lower break frequency) to one tenth of the way between ω_0 and $\omega_0\cdot 10$ (at the higher frequency).

Phase of Underdamped (Complex) Poles: Follow the low frequency asymptote at 0° *until*

$$\omega = \frac{\omega_0}{10^\zeta}$$

then decrease linearly to meet the high frequency asymptote at -180° at

$$\omega = \omega_0 \cdot 10^\zeta$$

Other magnitude and phase approximations (along with exact expressions) are given here.

Key Concept: Bode Plot for Complex Conjugate Poles

- For the magnitude plot of complex conjugate poles draw a 0 dB at low frequencies, go through a peak of height,

  $$|H(j\omega_0)| \approx \frac{1}{2\zeta}, \quad |H(j\omega_0)|_{dB} \approx -20 \cdot \log_{10}(2\zeta)$$

  at the break frequency and then drop at 40 dB per decade (i.e., the slope is -40 dB/decade). The high frequency asymptote goes through the break frequency. Note that in this approximation the peak only exists for

  $$0 < \zeta < 0.5$$

- To draw the phase plot simply follow low frequency asymptote at 0° until

  $$\omega = \frac{\omega_0}{10^\zeta} = \omega_0 \cdot 10^{-\zeta}$$

  then decrease linearly to meet the high frequency asymptote at -180° at

  $$\omega = \omega_0 \cdot 10^\zeta$$

  If $\zeta<0.02$, the approximation can be simply a vertical line at the break frequency.

- Note that the *shape* of the graphs (magnitude peak height, steepness of phase transition) are determined solely by ζ, and the frequency at which the magnitude peak and phase transition occur are determined solely by ω_0.

Note: Other magnitude and phase approximations (along with exact expressions) are given here. The analysis given above assumes the ζ is positive. For negative ζ see here

Interactive Demo

A Complex Conjugate Pair of Zeros

Not surprisingly a complex pair of zeros yields results similar to that for a complex pair of poles. The magnitude and phase plots for the complex zero are the mirror image (around 0dB for magnitude and around 0° for phase) of those for the complex pole. Therefore, the magnitude has a dip instead of a peak, the magnitude increases above the break frequency and the phase increases rather than decreasing. The results will not be derived here, but closely follow those for complex poles.

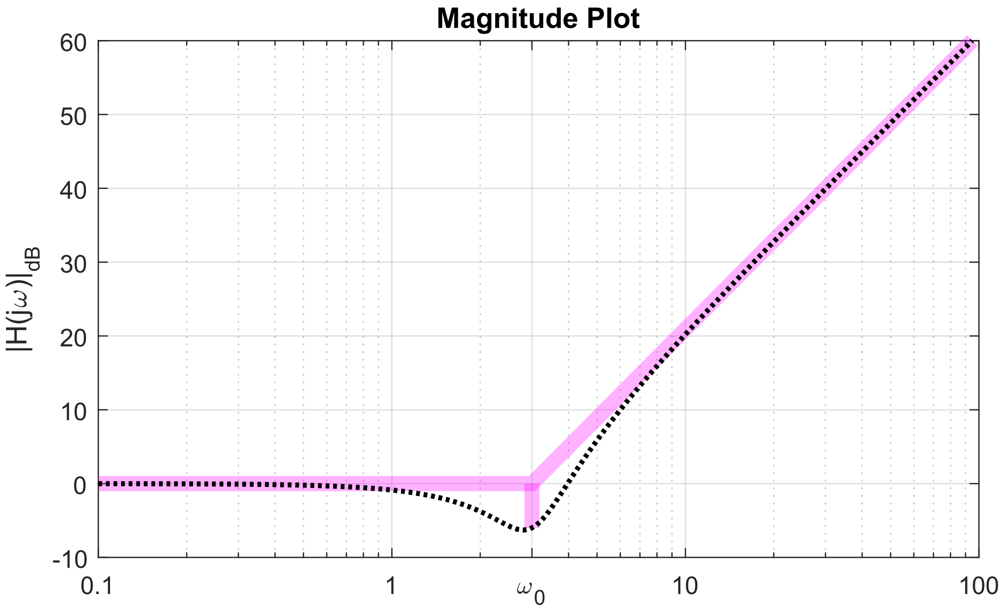
Note: The analysis given below assumes the ζ is positive. For negative ζ see here

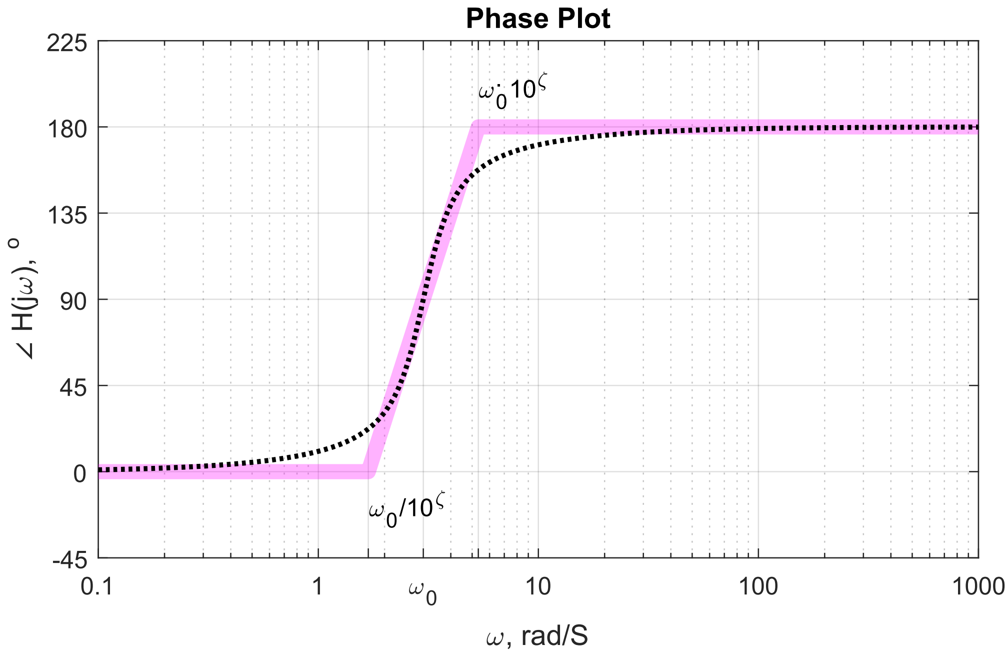
Example: Complex Conjugate Zero

The graph below corresponds to a complex conjugate zero with ω_0=3, ζ=0.25

$$H(s)=\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1$$

The dip in the magnitude plot will have a magnitude of 0.5 or -6 dB. The break frequencies for the phase are at $\omega=\omega_0/10^{\zeta}$=1.7 rad/sec and $\omega=\omega_0\cdot 10^{\zeta}$=5.3 rad/sec.





Key Concept: Bode Plot of Complex Conjugate Zeros

- The plots for a complex conjugate pair of zeros are very much like those for the poles but mirrored about 0dB or 0°.
- For the magnitude plot of complex conjugate zeros draw a 0 dB at low frequencies, go through a dip of magnitude:

$$|H(j\omega_0)| \approx 2\zeta, \quad |H(j\omega_0)|_{dB} \approx 20 \cdot \log_{10}(2\zeta)$$

  at the break frequency and then rise at +40 dB per decade (i.e., the slope is +40 dB/decade). The high frequency asymptote goes through the break frequency. Note that the peak only exists for

$$0 < \zeta < 0.5$$

- To draw the phase plot simply follow low frequency asymptote at 0° until

$$\omega = \frac{\omega_0}{10^{\zeta}} = \omega_0 \cdot 10^{-\zeta}$$

  then increase linearly to meet the high frequency asymptote at 180° at

$$\omega = \omega_0 \cdot 10^{\zeta}$$

- Note that the *shape* of the graphs (magnitude peak height, steepness of phase transition) are determined solely by ζ, and the frequency at which the magnitude peak and phase transition occur are determined solely by ω_0.

Note: Other magnitude and phase approximations (along with exact expressions) are given here.
The analysis given below assumes the ζ is positive. For negative ζ see here.

Interactive Demo

Non-Minimum Phase Systems

All of the examples above are for minimum phase systems. These systems have poles and zeros that do not have positive real parts. For example the term (s+2) is zero when s=-2, so it has a negative real root. First order poles and zeros have negative real roots if ω_0 is positive. Second order poles and zeros have negative real roots if ζ is positive. The magnitude plots for these systems remain unchanged, but the phase plots are inverted. See here for discussion.

Interactive Demos:

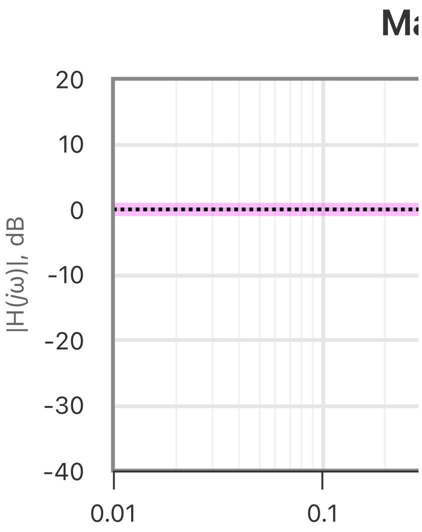
Below you will find interactive demos that show how to draw the asymptotic approximation for a constant, a first order pole and zero, and a second order (underdamped) pole and zero. Note there is no demo for a pole or zero at the origin because these are always drawn in exactly the same way; there are no variable parameters (i.e., ω_0 or ζ).

Interactive Demo: Bode Plot of Constant Term

This demonstration shows how the gain term affects a Bode plot. To run the demonstration either enter the value of K, or |K| expressed in dB, in one of the text boxes below. If you enter |K| in dB, then the sign of K is unchanged from its current value. You can also set |K| and ∠K by either clicking and dragging the horizontal lines on the graphs themselve. The magnitude of K must be between 0.01 and 100 (-40dB and +40dB). The phase of K (∠K) can only be 0° (for a positive value of K) or ±180° (for negative K).

Enter a value for gain, K: 1.00 ,

or enter |K| expressed in dB: 0.00 dB.

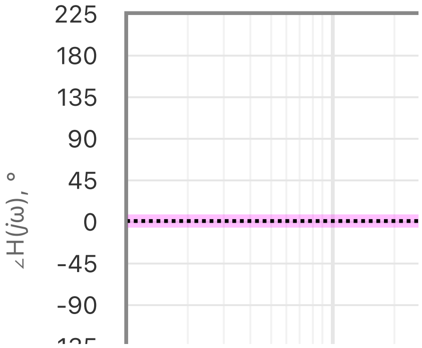


K = 1.00 so the value of

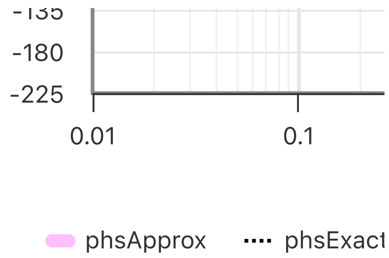
$K_{dB} = 20 \cdot \log_{10}(|K|) = 20 \cdot \log_{10}(|1.00|) = 0.00$.

Or, given that $K_{dB} = 0.00$, $|K| = 10^{Kdb/20} = 10^{0.00/20} = 1$.

The sign of K depends on phase, in this case K is positive.and phase = 0°.



Note that for the case of a constant term, the approximate (magenta line) and exact (dotted black line) representations of magnitude and phase are equal.

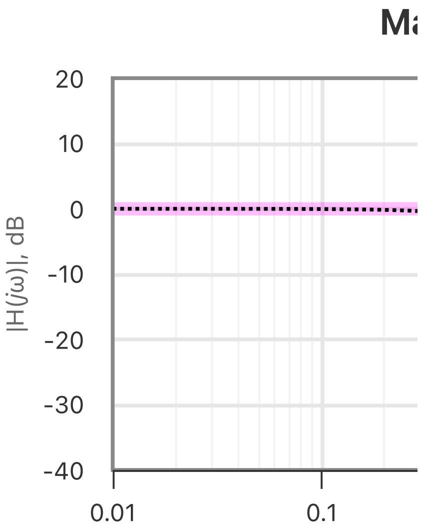


Interactive Demo: Bode Plot of a Real Pole

This demonstration shows how a first order pole expressed as:

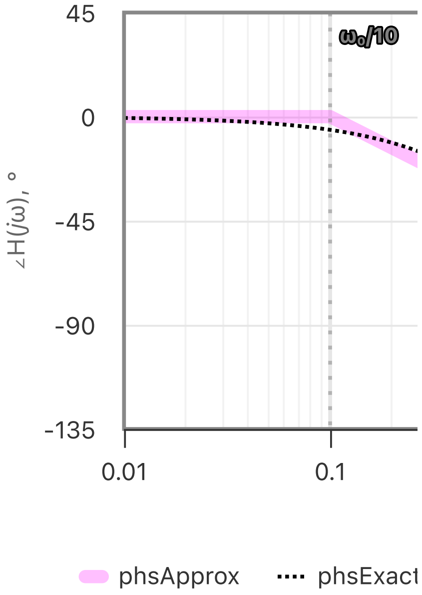
$$H(s)=\frac{1}{1+\frac{s}{\omega_0}}=\frac{1}{1+j\frac{\omega}{\omega_0}},$$

is displayed on a Bode plot. To change the value of ω_0, you can either change the value in the text box, below, or drag the vertical line showing ω_0 on the graphs to the right. The exact values of magnitude and phase are shown as black dotted lines and the asymptotic approximations are shown with a thick magenta line. The value of ω_0 is constrained such that $0.1 \leq \omega_0 \leq 10$ rad/second.



Enter a value for ω_o: 1.000

Asymptotic Magnitude: The asymptotic magnitude plot starts (at low frequencies) at 0 dB and stays at that level until it gets to ω_0. At that point the gain starts dropping with a slope of -20 dB/decade.



Asymptotic Phase: The asymptotic phase plot starts (at low frequencies) at 0° and stays at that level until it gets to $0.1\cdot\omega_0$ (0.1 rad/sec). At that point the phase starts dropping at -45°/decade until it gets to -90° at $10\cdot\omega_0$ (10 rad/sec), at which point it becomes constant at -90° for high frequencies. Phase goes through -45° at $\omega=\omega_0$.

$\omega_0/10$	ω_0	$10\cdot\omega_0$
0.10	1.00	10.00

Interactive Demo: Bode Plot of a Real zero

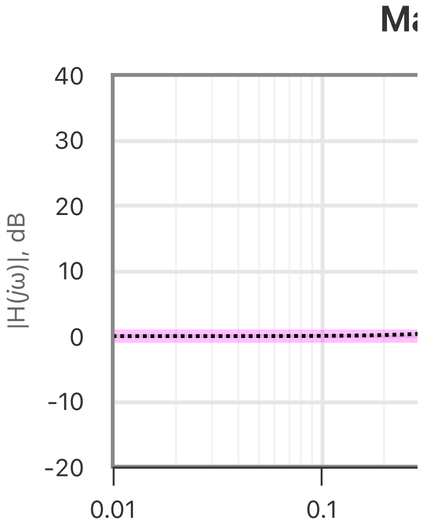
This demonstration shows how a first order zero expressed as:

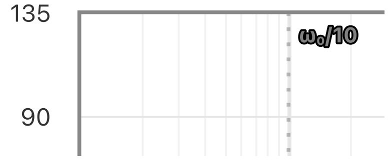
$$H(s) = 1 + \frac{s}{\omega_0} = 1 + j\frac{\omega}{\omega_0},$$

is displayed on a Bode plot. To change the value of ω_0, you can either change the value in the text box, below, or drag the vertical line showing ω_0 on the graphs to the right. The exact values of magnitude and phase are shown as black dotted lines and the asymptotic approximations are shown with a thick magenta line. The value of ω_0 is constrained such that $0.1 \leq \omega_0 \leq 10$ rad/second.

Enter a value for ω_o: 1.000

Asymptotic Magnitude: The asymptotic magnitude plot starts (at low frequencies) at 0 dB and stays at that





Rules for Constructing Bode Diagrams

Overview | Freq Domain | Asymptotic plots | Making Plot | Examples | Drawing Tool
BodePlotGui | Rules Table | Printable

This document will discuss how to actually draw Bode diagrams. It consists mostly of examples.

Key Concept -
To draw Bode diagram there are four steps:

1. Rewrite the transfer function in proper form.
2. Separate the transfer function into its constituent parts.
3. Draw the Bode diagram for each part.
4. Draw the overall Bode diagram by adding up the results from part 3.

---

1. Rewrite the transfer function in proper form.

A transfer function is normally of the form:

$$H(s) = K\frac{\sum_{i=0}^{m} b_m s^m}{\sum_{j=0}^{n} a_n s^n}$$

As discussed in the previous document, we would like to rewrite this so the lowest order term in the numerator and denominator are both unity.

Some examples will clarify:

Example 1

$$H(s) = 30\frac{s+10}{s^2+3s+50} = 30\frac{10}{50}\frac{\frac{s}{10}+1}{\frac{s^2}{50}+\frac{3}{50}s+1} = 6\frac{\frac{s}{10}+1}{\frac{s^2}{50}+\frac{3}{50}s+1}$$

Note that the final result has the lowest (zero) order power of numerator and denominator polynomial equal to unity.

Example 2

$$H(s) = 30\frac{5s}{s^2+3s+50} = 30\frac{5}{50}\frac{\frac{s}{1}}{\frac{s^2}{50}+\frac{3}{50}s+1} = 3\frac{\frac{s}{1}}{\frac{s^2}{50}+\frac{3}{50}s+1}$$

Note that in this example, the lowest power in the numerator was 1.

Example 3

$$H(s) = 30\frac{s+10}{(s+3)(s+50)} = 30\frac{10}{3\cdot 50}\frac{\frac{s}{10}+1}{\left(\frac{s}{3}+1\right)\left(\frac{s}{50}+1\right)}$$

$$= 2\frac{\frac{s}{10}+1}{\left(\frac{s}{3}+1\right)\left(\frac{s}{50}+1\right)}$$

In this example the denominator was already factored. In cases like this, each factored term needs to have unity as the lowest order power of s (zero in this case).

---

2. Separate the transfer function into its constituent parts.

The next step is to split up the function into its constituent parts. There are seven types of parts:

1. A constant
2. Poles at the origin
3. Zeros at the origin
4. Real Poles
5. Real Zeros
6. Complex conjugate poles
7. Complex conjugate zeros

We can use the examples above to demonstrate again.

Example 1

$$H(s) = 30\frac{s+10}{s^2+3s+50} = 30\frac{10}{50}\frac{\frac{s}{10}+1}{\frac{s^2}{50}+\frac{3}{50}s+1} = 6\frac{\frac{s}{10}+1}{\frac{s^2}{50}+\frac{3}{50}s+1}$$

This function has

- a constant of 6,
- a zero at s=-10,
- and complex conjugate poles at the roots of $s^2+3s+50$.

The complex conjugate poles are at s=-1.5 ± *j*6.9 (where *j*=sqrt(-1)). A more common (and useful for our purposes) way to express this is to use the standard notation for a second order polynomial

$$\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right) + 1$$

In this case

$$\omega_0 = \sqrt{50} = 7.07, \quad \zeta = \frac{3\sqrt{50}}{2 \cdot 50} = 0.21$$

Example 2

$$H(s) = 30\frac{5s}{s^2 + 3s + 50} = 30\frac{5}{50}\frac{\frac{s}{1}}{\frac{s^2}{50} + \frac{3}{50}s + 1} = 3\frac{\frac{s}{1}}{\frac{s^2}{50} + \frac{3}{50}s + 1}$$

This function has

- a constant of 3,
- a zero at the origin,
- and complex conjugate poles at the roots of $s^2+3s+50$, in other words

$$\omega_0 = \sqrt{50} = 7.07, \quad \zeta = \frac{3\sqrt{50}}{2 \cdot 50} = 0.21$$

Example 3

$$H(s) = 30\frac{s + 10}{(s + 3)(s + 50)} = 2\frac{\frac{s}{10} + 1}{\left(\frac{s}{3} + 1\right)\left(\frac{s}{50} + 1\right)}$$

This function has

- a constant of 2,
- a zero at s=-10, and
- poles at s=-3 and s=-50.

---

3. Draw the Bode diagram for each part.

The rules for drawing the Bode diagram for each part are summarized on a separate page. Examples of each are given later.

---

4. Draw the overall Bode diagram by adding up the results from step 3.

After the individual terms are drawn, it is a simple matter to add them together. See examples, below.

Examples: Draw Bode Diagrams for the following transfer functions

These examples are compiled on the next page.

Example 1

A simple pole

$$H(s)=\frac{100}{s+30}$$

Full Solution

Example 2

Multiple poles and zeros

$$H(s)=100\frac{(s+1)}{(s+10)(s+100)}=100\frac{(s+1)}{s^2+110s+1000}$$

Full Solution

Example 3

A pole at the origin and poles and zeros

$$H(s)=10\frac{s+10}{s^2+3s}$$

Full Solution

Example 4

Repeated poles, a zero at the origin, and a negative constant

$$H(s)=-100\frac{s}{s^3+12s^2+21s+10}$$

Full Solution

Example 5

Complex conjugate poles

$$H(s)=30\frac{s+10}{s^2+3s+50}$$

Full Solution

Example 6

A complicated function

Bode Plot Examples

Overview | Freq Domain | Asymptotic plots | Making Plot | Examples | Drawing Tool | BodePlotGui | Rules Table | Printable

Several examples of the construction of Bode plots are included here; click on the transfer function in the table below to jump to that example.

Examples (Click on Transfer Function)

1 $\frac{100}{s+30}$ (a real pole)	**2** $100\frac{s+1}{s^2+110s+1000}$ (real poles and zeros)	**3** $10\frac{s+10}{s^2+3s}$ (pole at origin)	**4** $-100\frac{s}{s^3+12s^2+21s+10}$ (repeated real poles, negative constant)	**5** $30\frac{s+10}{s^2+3s+50}$ (complex conj. poles)	**6** $4\frac{s^2+s+25}{s^3+100s^2}$ (multiple poles at origin, complex conj zeros)

References

Rules for Drawing Bode Diagrams

The table below summarizes what to do for each type of term in a Bode Plot. This is also available as a Word Document or PDF.

The table assumes ω_0>0. If ω_0<0, magnitude is unchanged, but phase is reversed.

Term	Magnitude	Phase
Constant: K	20log$_{10}$(IKI)	K>0: 0° K<0: ±180°
Pole at Origin (Integrator) $\frac{1}{s}$	-20 dB/decade passing through 0 dB at ω=1	-90°
Zero at Origin (Differentiator) s	+20 dB/decade passing through 0 dB at ω=1 *(Mirror image, around x axis,of Integrator)*	+90° *(Mirror image, around x axis, of Integrator about)*
Real Pole $\dfrac{1}{\frac{s}{\omega_0}+1}$	1. Draw low frequency asymptote at 0 dB. 2. Draw high frequency asymptote at -20 dB/decade. 3. Connect lines at ω_0.	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at -90° 3. Connect with a straight line from 0.1·ω_0 to 10·ω_0
Real Zero $\frac{s}{\omega_0}+1$	1. Draw low frequency asymptote at 0 dB. 2. Draw high frequency asymptote at +20 dB/decade. 3. Connect lines at ω_0. *(Mirror image, around x-axis, of Real Pole)*	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at +90° 3. Connect with a straight line from 0.1·ω_0 to 10·ω_0 *(Mirror image, around x-axis, of Real Pole about 0°)*
Underdamped Poles (Complex conjugate poles) $\dfrac{1}{\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1}$ $0<\zeta<1$	1. Draw low frequency asymptote at 0 dB. 2. Draw high frequency asymptote at -40 dB/decade. 3. Connect lines at ω_0. 4. If ζ<0.5, then draw peak at ω_0 with amplitude IH(jω_0)I=-20·log$_{10}$(2ζ), else don't draw peak	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at -180° 3. Connect with straight line from $\omega=\dfrac{\omega_0}{10^\zeta}$ to $\omega_0\cdot 10^\zeta$ *You can also look in a*

	(it is very small).	*textbook for examples*
Underdamped Zeros (Complex conjugate zeros) $\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right) + 1$ $0 < \zeta < 1$	1. Draw low frequency asymptote at 0 dB. 2. Draw high frequency asymptote at +40 dB/decade. 3. Connect lines at ω_0. 4. If $\zeta<0.5$, then draw peak at ω_0 with amplitude $\lvert H(j\omega_0)\rvert=+20\cdot\log_{10}(2\zeta)$, else don't draw peak (it is very small). *(Mirror image, around x-axis, of Underdamped Pole)*	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at +180° 3. Connect with straight line from $\omega = \frac{\omega_0}{10^\zeta}$ to $\omega_0 \cdot 10^\zeta$ *You can also look in a textbook for examples.* *(Mirror image, around x-axis, of Underdamped Pole)*

For multiple order poles and zeros, simply multiply the slope of the magnitude plot

BodePlotGui: A Tool for Generating Asymptotic Bode Diagrams

Overview | Freq Domain | Asymptotic plots | Making Plot | Examples | Drawing Tool | BodePlotGui | Rules Table | Printable

BodePlotGui is a graphical user interface written in the MATLAB® programming language. It takes a transfer function and splits it into its constituent elements, then draws the piecewise linear asymptotic approximation for each element. It is hoped that the BodePlotGui program will be a versatile program for teaching and learning the construction of Bode diagrams from piecewise linear approximations.

Files for the program are found here.

Note: the MATLAB GUI doesn't display well on all devices (some elements of the GUI may not show up). If you have this problem, simply run the MATLAB command "`guide`" and open the file*BodePlotGui.fig*. You can edit the size and layout of the GUI for your machine. Save it, and then rerun the *BodePlotGui.m* file.

I have stopped working on BodePlotGui and have developed a similar tool in JavaScript to make it more accessible (see the "Drawing Tool" tab, above). While MATLAB is extremely powerful, it is also very expensive.

Use of program.

A Simple Example.

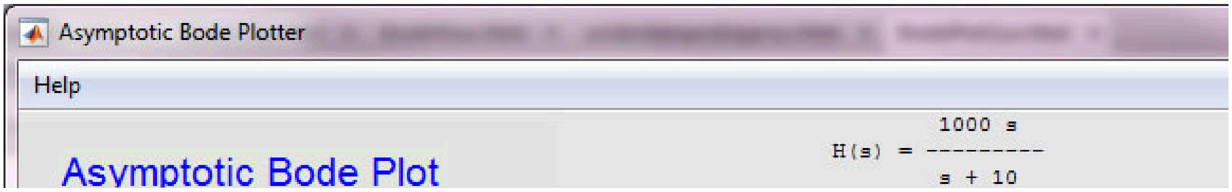
Consider the transfer function:

$$H(s) = 1000\frac{s}{s+10} = 100\frac{s}{1+\dfrac{s}{10}}$$

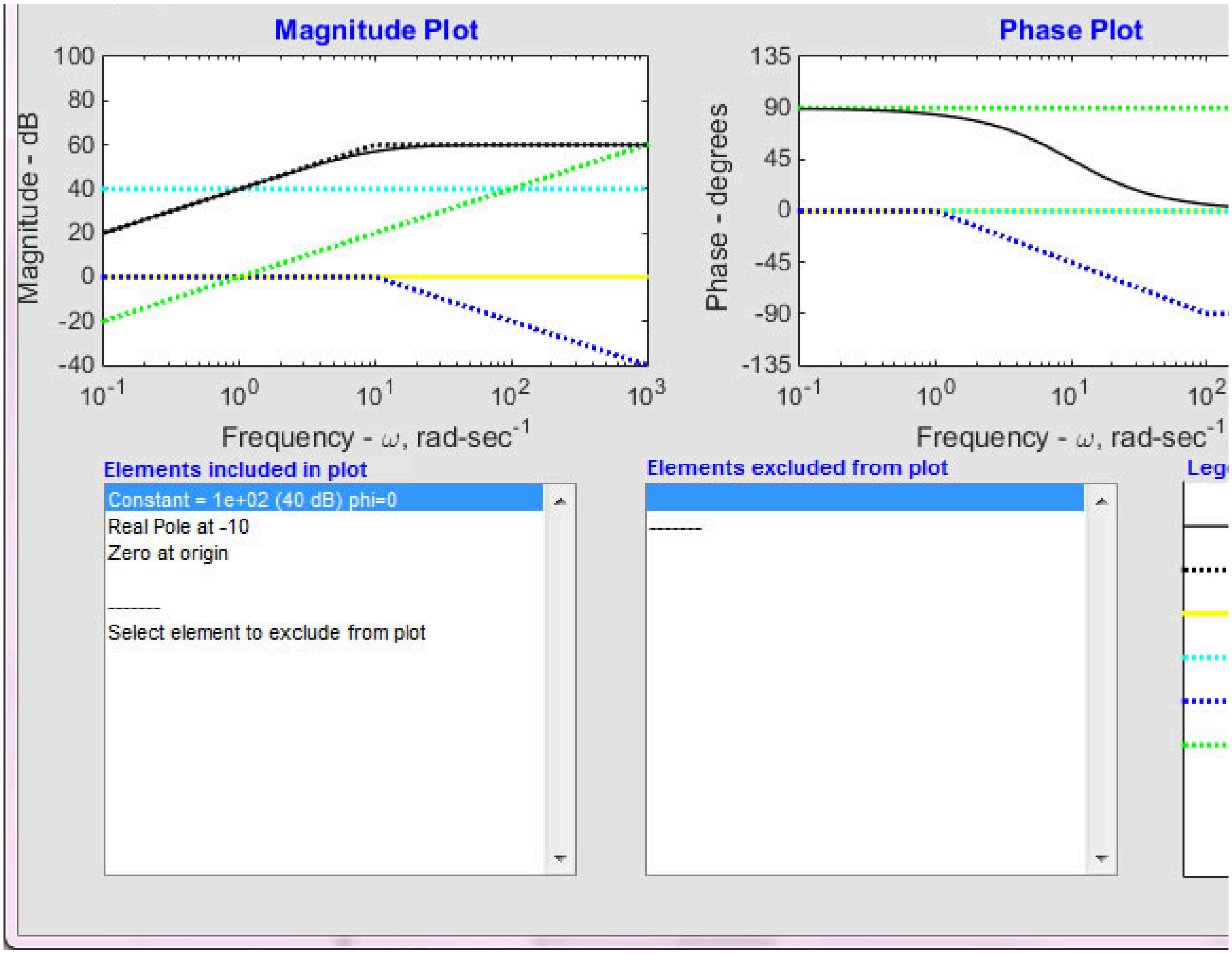
This function has three terms to be considered when constructing a Bode diagram, a constant (100), a pole at ω=10 rad/sec, and a zero at the origin. The following MATLAB® commands begin execution of the GUI:

```
>>MySys=tf(1000*[1 0],[1 10]);    %define Xfer function
>>BodePlotGui(MySys)          %Invoke GUI
```

The GUI generates a window as shown below.



```
        1000 s
H(s) = ---------
        s + 10
```



Elements included in plot

Constant = 1e+02 (40 dB) phi=0
Real Pole at -10
Zero at origin

Select element to exclude from plot

Elements excluded from plot

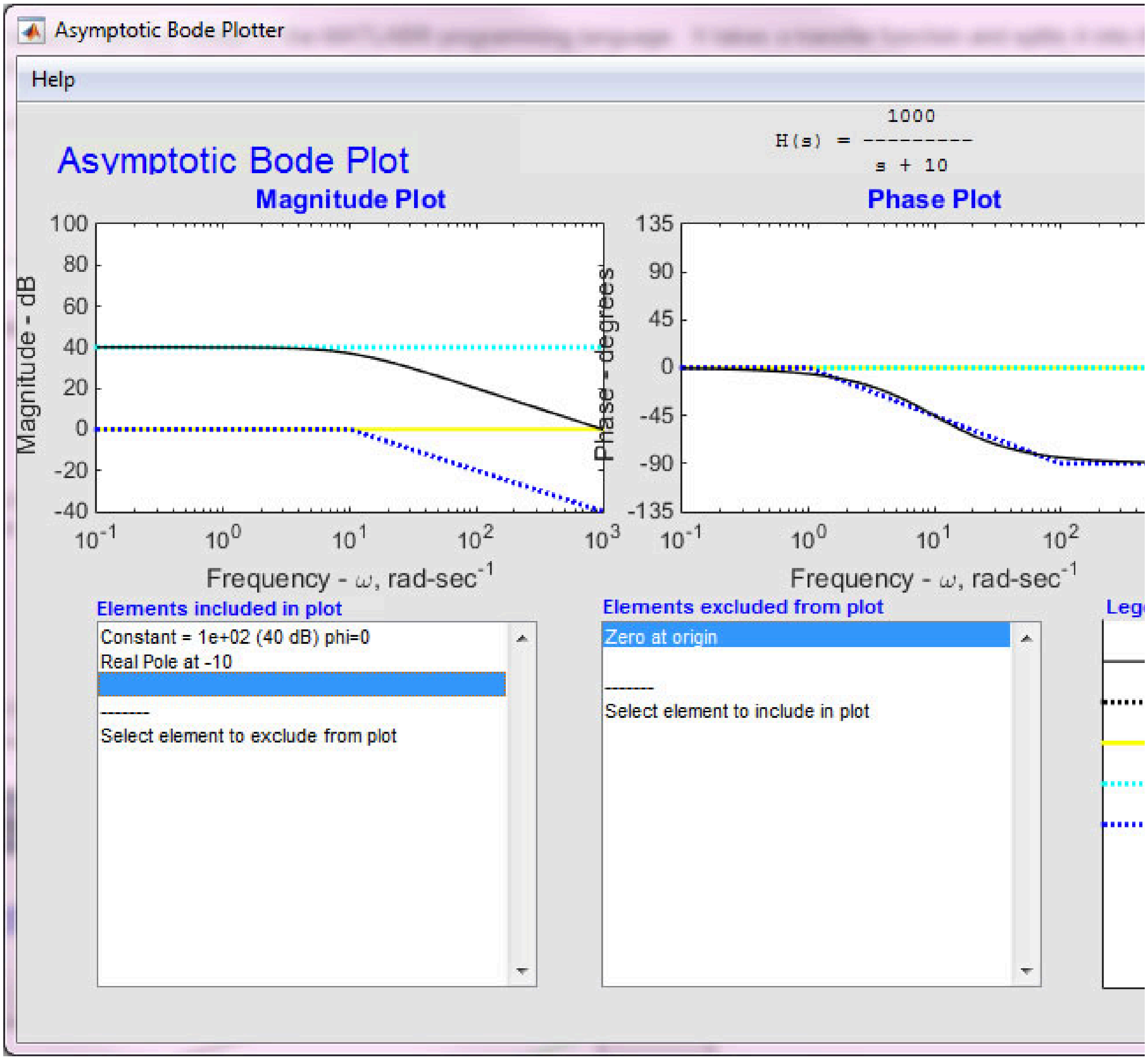
Starting in the upper left and going counterclockwise, the windows show:

1. The magnitude plot, both the piecewise linear approximation for all three terms as well as the asymptotic plot for the complete transfer function and the exact Bode diagram for magnitude. Also shown is a zero reference line.
2. The phase plot.
3. A list of the systems in the user workspace.
4. Several checkboxes that let the user format the image. In particular there is a check-box that determines whether or not to display the asymptotic plot for the complete transfer function; sometimes it gets in the way of seeing the other plots, so you may want to hide it.
5. The legend identifying individual terms on the plot.
6. A box that shows elements excluded from the plot. This box is empty in this display because the diagram displays all three elements of the transfer function.
7. A 'Legend' box that shows elements displayed in the plot.
8. Several check-boxes that allow the user to display how the plots are displayed
9. Also in the upper left is a "Help" tab.

Also shown in the upper right hand corner is the transfer function, H(s).

Modifying what is displayed

The function displayed can be manipulated term by term to illustrate the effect of each term. For example, the zero at the origin can be excluded simply by clicking on it in the lower left hand box. The figure below shows the result in the lower left hand box. The figure below shows the result.



$$H(s) = \frac{1000}{s + 10}$$

Elements included in plot:

Constant = 1e+02 (40 dB) phi=0
Real Pole at -10

Select element to exclude from plot

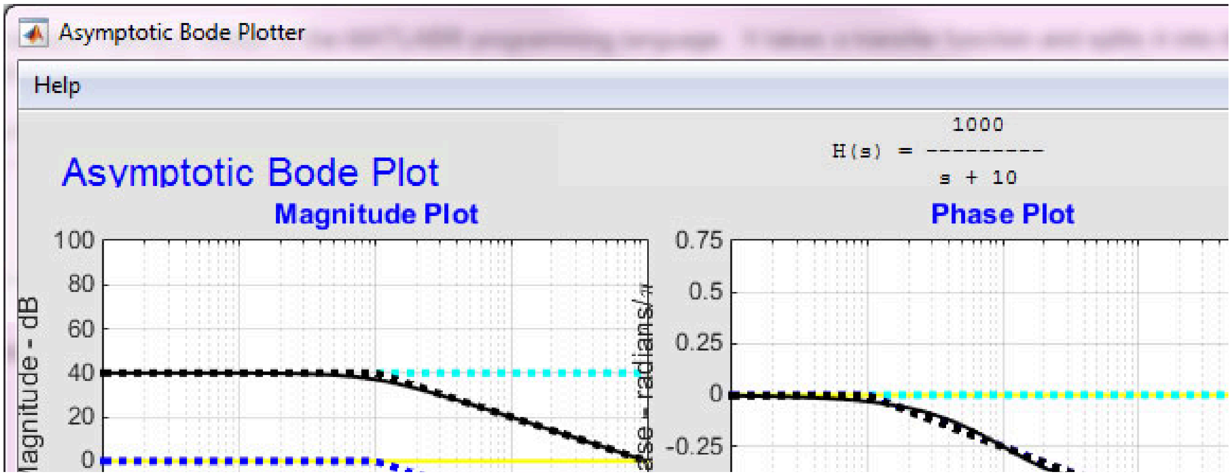
Elements excluded from plot:

Zero at origin

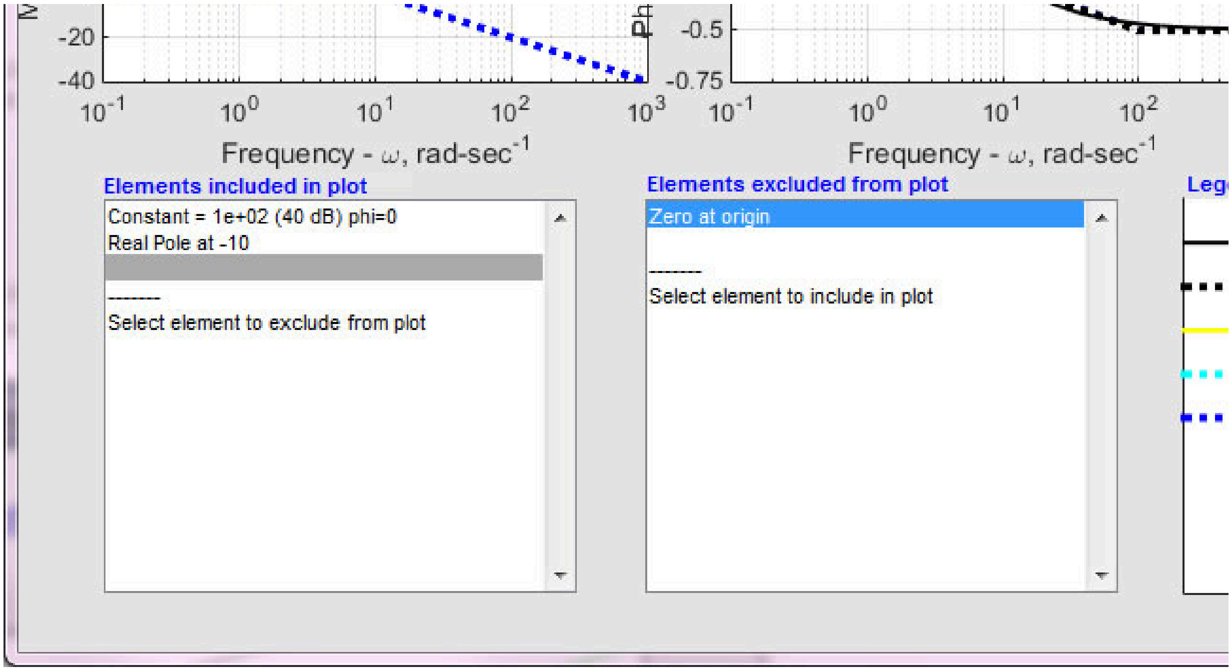
Select element to include in plot

Note that the zero at the origin is no longer included in the plot. Each term can be likewise included or excluded by simply clicking on it.

The next plot shows the plot modified to have thicker lines, a grid, phase in radians and with the asymptotic plot of the complete transfer function. In the previous graph, the phase of the asymptotic plot obscured that of the real pole; this is an example when it might be convenient not to show the asymptotic approximation.



$$H(s) = \frac{1000}{s + 10}$$



Elements included in plot

Constant = 1e+02 (40 dB) phi=0
Real Pole at -10

Select element to exclude from plot

Elements excluded from plot

Zero at origin

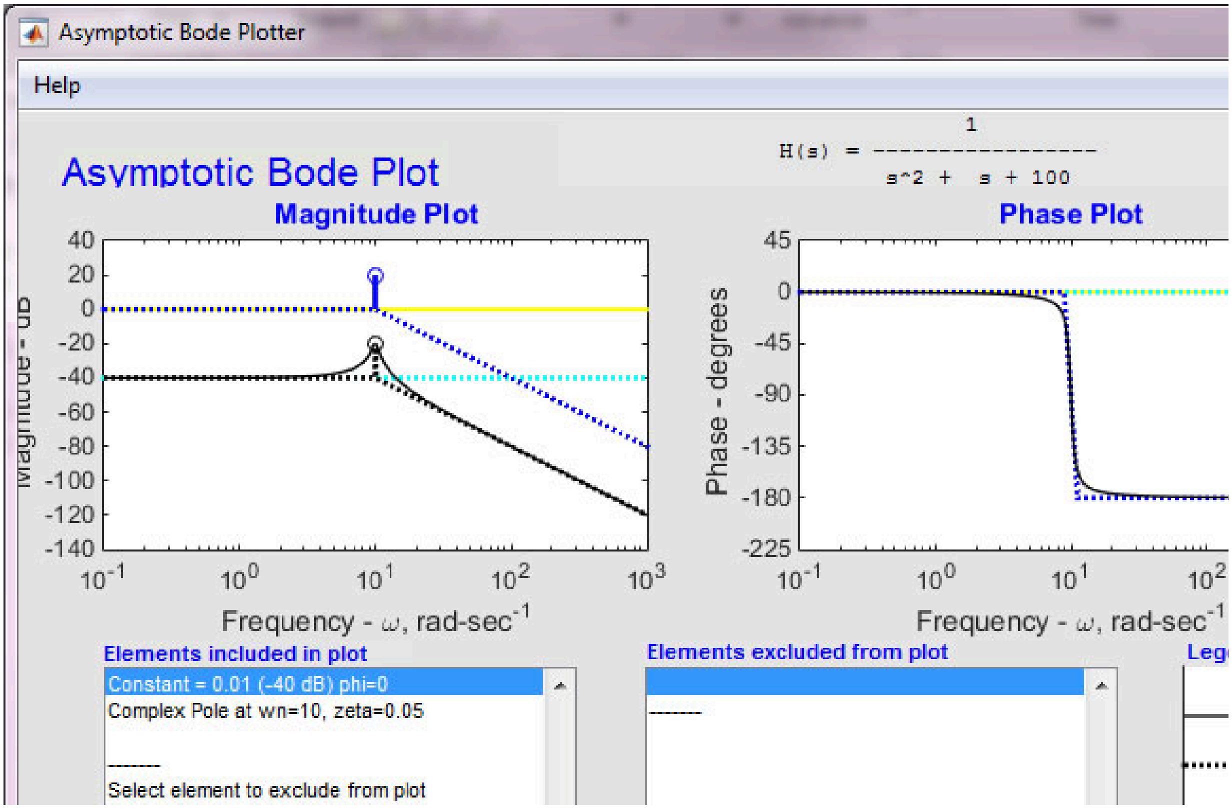
Select element to include in plot

Underdamped terms

Underdamped poles (and zeros) present a difficulty because they cause a peak (dip) in the magnitude plot. The program show this with a simple circle showing the peak height. For example the transfer function

$$H(s) = \frac{1}{s^2 + s + 100}$$

yields the output shown below. The peak due to the underdamped pole is clearly shown.



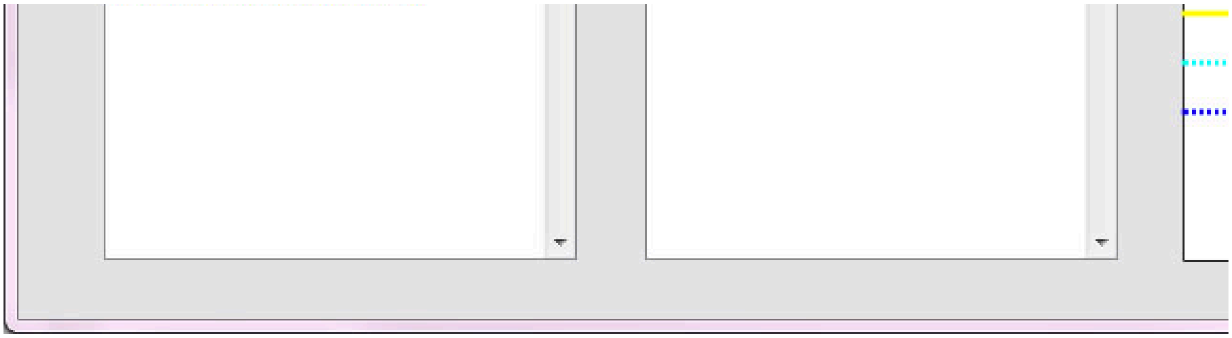
```
                 1
H(s) = ----------------
        s^2 +   s + 100
```

Elements included in plot

Constant = 0.01 (-40 dB) phi=0
Complex Pole at wn=10, zeta=0.05

Select element to exclude from plot

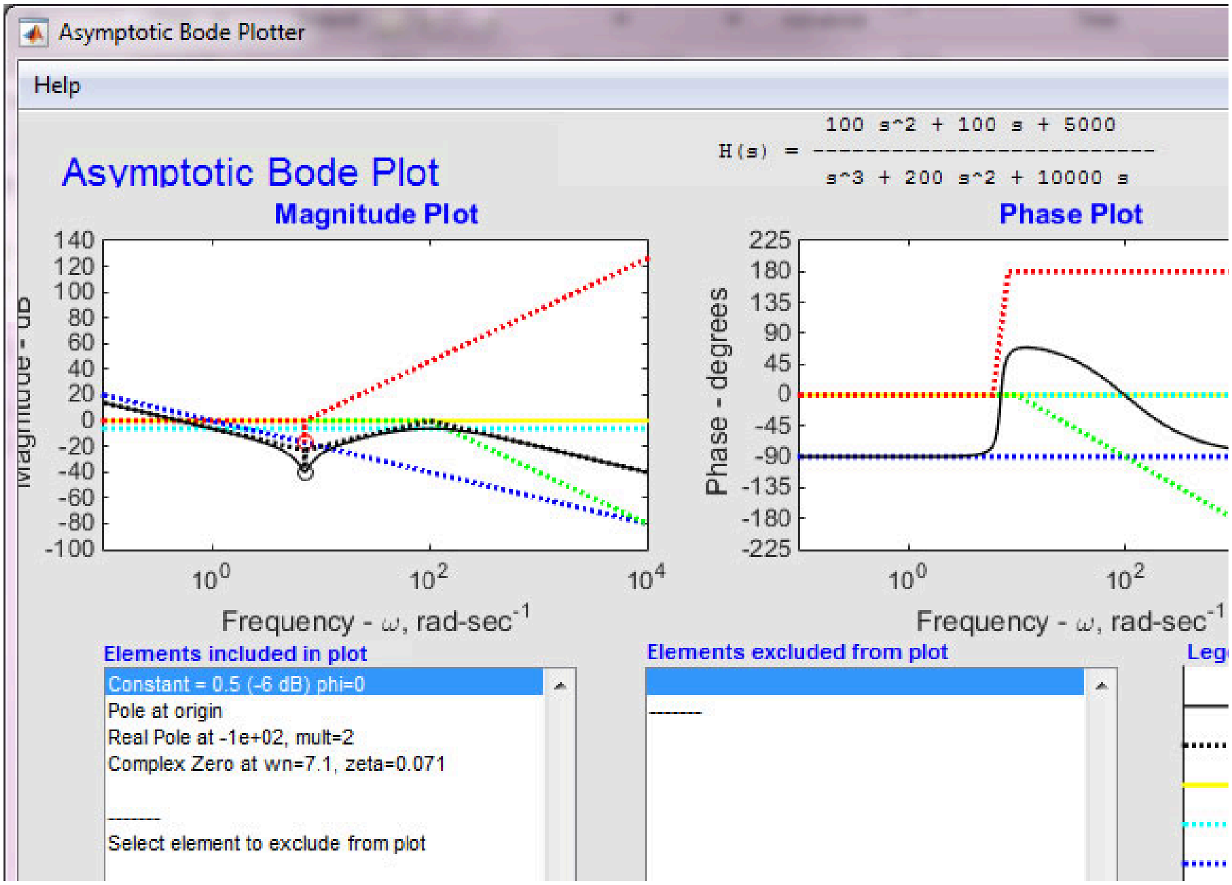
Elements excluded from plot



A more complicated example

The example below is more complicated. It shows underdamped terms, repeated poles, and a pole at the origin.

$$H(s) = 100\frac{s^2+s+50}{s^3+200s^2+10000s} = 100\frac{s^2+s+50}{s(s+100)^2}$$



```
        100 s^2 + 100 s + 5000
H(s) = --------------------------
        s^3 + 200 s^2 + 10000 s
```

Elements included in plot

Constant = 0.5 (-6 dB) phi=0
Pole at origin
Real Pole at -1e+02, mult=2
Complex Zero at wn=7.1, zeta=0.071

Select element to exclude from plot

Elements excluded from plot

Make your own Bode plot paper

The code for BodePaper.m is available at https://github.com/echeever/BodePlotGui

When making Bode plots one needs two pieces of semi-logarithmic paper, one for the magnitude plot and one for the phase. The program described here, BodePaper.m, can be used to make paper. Download it and save it so that MatLab can find it (from the Matlab menu you can go to *File→Set Path* and include the directory where you stored the BodePaper.m file.) . There is also a fine in the repository called BodeMagPaper.m that creates only a magnitude plot.

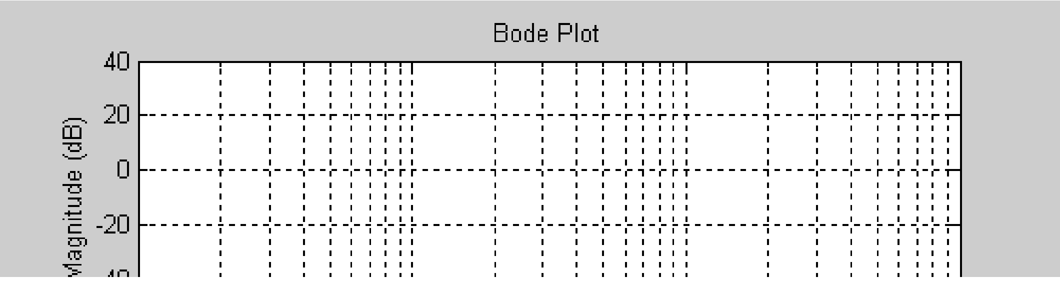
The syntax for calling is given by the function's help file.

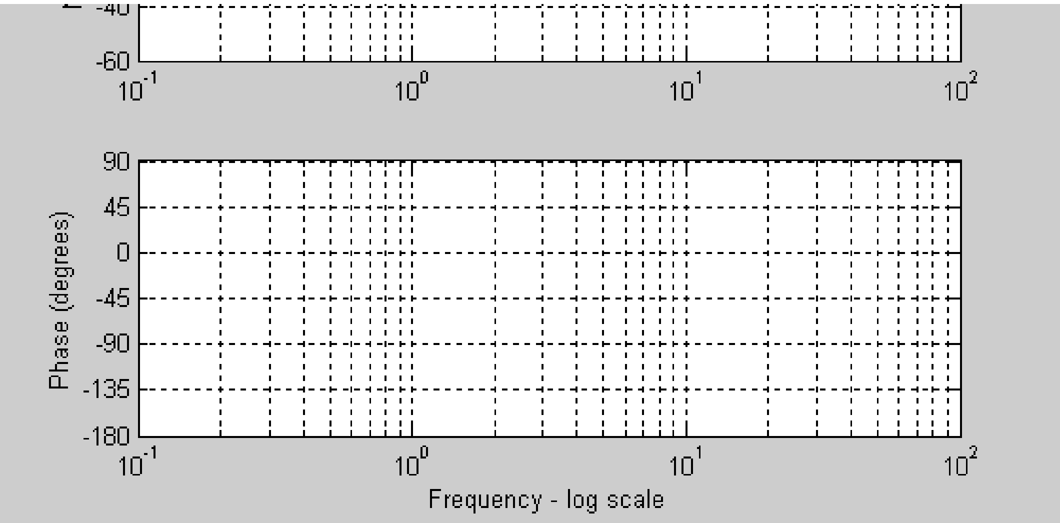
```
>> help BodePaper

BodePaper is Matlab code to generate graph paper for Bode
two semilog graphs for making Bode plots. The top plot is
units on the vertical axis is set to dB. The bottom plot s
units on the phase plot can be radians or degrees, at the
user. The default is degrees.

The correct calling syntax is:
BodePaper(om_lo, om_hi, dB_lo, dB_hi, ph_lo, ph_hi, UseRad
    om_lo the low end of the frequency scale. This can be
        rad/sec or Hz. No units are displayed on the graph
    om_hi the high end of the frequency scale.
    dB_lo the bottom end of the dB scale.
    dB_hi the top end of the dB scale.
    ph_lo the bottom end of the phase scale.
    ph_hi the top end of the phase scale.
    UseRad an optional argument. If this argument is non-z
        on the phase plot are radians. If this argument is
        or set to zero, the units are degrees.
```

To make paper that goes from 0.1 Hz to 100 Hz, with the magnitude scale going from -60 to 40 dB and the phase from -180 to 90 degrees, the function call would be

```
>> BodePaper(0.1,100,-60,40,-180,90)
```

and the paper looks like:

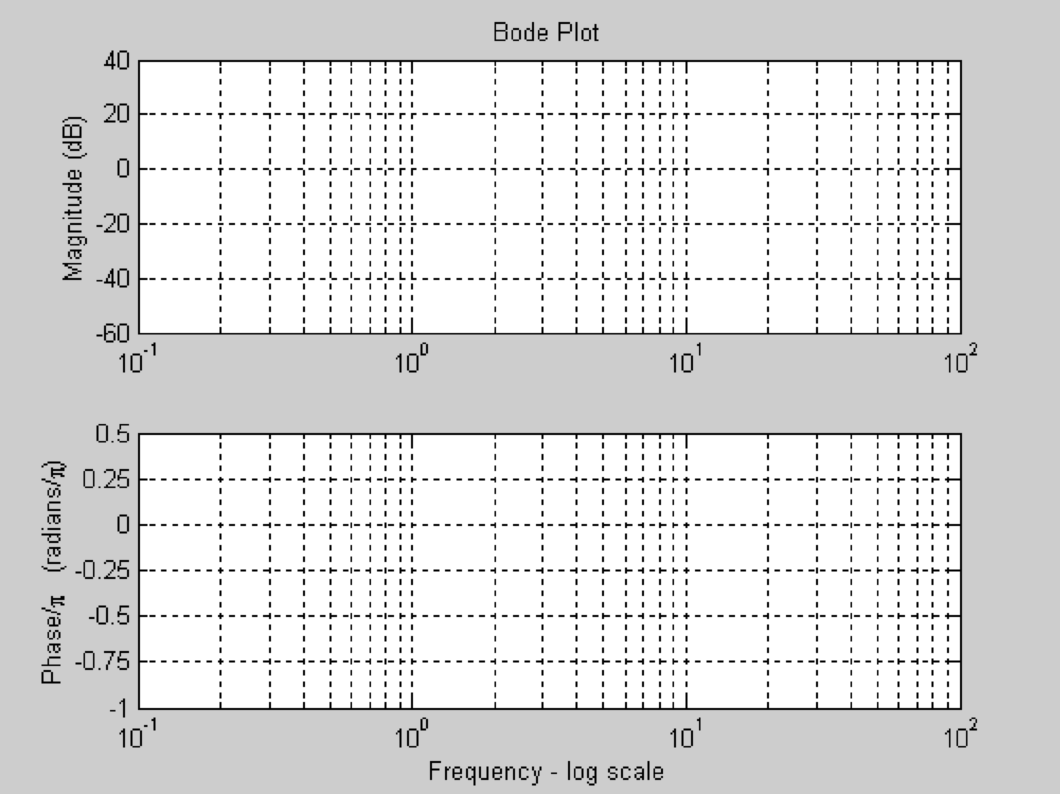




To change the units on phase the function call would be:

```
BodePaper(0.1,100,-60,40,-pi,pi/2,1)
```

and the paper now looks like this:



References

Refere

Refere