MEASURING MOTOR PARAMETERS **File: Motor parameters**

These are the motor parameters that are needed:

Motor voltage constant	K_e (volts-sec/rad)
Motor torque constant	K_T (lb-in/amp)
Motor resistance	R_a (ohms)
Motor inductance	L_a (Henries)
Motor inertia	J_m (lb-in-sec^2)
Load inertia reflected to the motor armature shaft	J_{load} (lb-in-sec^2)
Total inertia=J_m+J_{load}	J_{total}(lb-in-sec^2)

Note that the above values are stated for a single winding with dc motors, and are the phase values for a BLDC motor. Brushless dc motors (BLDC) are 3 phase synchronous motors used in a configuration to be treated as dc drives.

MOTOR RESISTANCE

For the winding resistance use an ohmmeter. For a dc motor measure the resistance between the 2 armature wires. If it is a WYE connected BLDC motor, the resistance is the line-to-line resistance. Thus divide the resistance$_{(l\text{-}l)}$ by 2 to get the phase resistance.

MOTOR VOLTAGE CONSTANT K_e

To measure the K_e of the motor, put the motor shaft in a lathe and rotate the shaft at some speed [rpm] such as 1000rpm. With a dc motor, use a dc voltmeter to measure the armature voltage. The K_e is then the voltage you read divided by the speed in rad/sec. Convert rpm to rad/sec as-

$$\frac{rev}{\min}\ x\ \frac{2\pi\ radians}{rev}\ x\ \frac{\min}{60\sec} = \left[\frac{rad}{\sec}\right]$$

$$K_e = \frac{Volts\,[v]}{speed\ [rad/\sec]}\ or\ \frac{Volts[v]}{speed[rpm]}$$

With a BLDC motor use an ac voltmeter to measure the voltage between any 2 wires of the 3 motor wires and then convert the line-to-line voltage to the phase voltage value by dividing the line-to-line voltage by $\sqrt{3}$ =1.73.

$$K_e(phase) = \frac{K_e(line-to-line)}{1.73}$$

MOTOR TORQUE CONSTANT K_T (for a BLDC MOTOR)

The motor torque constant (K_T) can be computed from the voltage constant (K_e) as-

$$K_T\left[\frac{lb-in}{amp}\right]=\frac{K_e}{0.00684}\left[\frac{volts_{(l-l)}}{rpm}\right]$$

The derivation for the above equation is-

Equate Electrical Power to Mechanical Power converted to Watts

$$\sqrt{3}\ x\ E\ x\ I=\frac{2\pi}{60}\ x\ N\ x\ \frac{T}{12}\ x1.356$$

where: T= lb-in

I= Amps

E= $\text{Volts}_{\text{rms(line-to-line)}}$

N= rpm

Rearranging terms and simplifying:

$$K_e=\frac{E}{N}=0.00684\frac{T}{I}$$

$$Where:\frac{E}{N}=K_e=\left(Back\,emf\ cons\tan t,\frac{Volts_{rms(l-l)}}{rpm}\right)$$

$$\frac{T}{I}=K_T=\left[Torque\ \ cons\tan t,\frac{lb-in}{A}\right]$$

Converting rpm to rad/sec-

$$K_e=\left[\frac{v\ x\ rev\ x\ 60\sec}{\frac{rev}{\min}\ 2\pi\ x\ \min}\right]=\left[\frac{v\ x\ 60}{\frac{rad}{\sec}\ x\ 2\pi}\right]=9.554\left[\frac{v_{(l-l)}}{\frac{rad}{\sec}}\right]$$

Thus

$$K_T=\frac{K_e\,[v-\sec/rad]}{0.00684\,x\ 9.554}$$

$$K_T\left[\frac{lb-in}{amp}\right]=15.3\ K_e\left[\frac{v_{(l-l)}}{\frac{rad}{\sec}}\right]$$

MOTOR TORQUE CONSTANT Kt (for a DC MOTOR)

$$E\,I = \frac{2\pi}{60} N \frac{T}{12} x1.356$$

$$E\,I = 0.011827\ N\,T$$

$$K_e = \frac{E}{N} = 0.011827\ \frac{T}{I}$$

$$Therefore: \quad K_T\left[\frac{lb-in}{amp}\right] = \frac{K_e}{0.011827}\left[\frac{V}{RPM}\right]$$

MOTOR INERTIA J_m

Motor rotor inertia can be measured by making an experiment. The inertia can be calculated from the equation-

$$Acceleration \quad torque[lb-in] = Inertia[lb-in-sec\hat{}2] \quad x\ acceleration\left[\frac{rad}{sec\hat{}2}\right]$$

Also (rearranging terms)

$$Inertia = \frac{accel\ torque}{acceleration} \quad [lb-in-sec\hat{}2]$$

DC MOTORS

To do this test it is necessary to measure the acceleration of the motor rotor and the acceleration torque of the motor rotor. These two parameters are described as follows:

ACCELERATION – This parameter is determined by putting a step in current into the motor winding to bring the motor up to rated speed. The motor will accelerate exponentially. The acceleration is a measure of the rate of change of velocity over a period of time. To make this test, a dc tachometer should be connected to the motor shaft. The output of the tachometer should be connected to a stripchart recorder. When a step input in current is applied to the motor winding, the chart recorder will plot the rate of change of the motor shaft velocity as a function of the tachometer output voltage. The tachometer calibration can be used to convert volts to rpm. The acceleration is therefore the change in velocity for the linear part of the exponential curve divided by the time elapsed for the detected rate of change in velocity. The resulting calculation of the acceleration must have the dimensions changed to be in units of rad/sec^2.

TORQUE- An ammeter must be inserted in series with the motor input winding. When the step in input current is applied to the motor input, the maximum value of current should be noted. This current must be converted to torque. The torque is equal to the maximum value of current observed multiplied by the motor torque constant. Torque [lb-in]= amps[a] x K_T [lb-in/a]. The inertia is then the acceleration torque divided by the acceleration as stated above.

BLDC MOTORS- The tests described thus far must be modified for a bldc motor. A bldc motor must be tested with its servo amplifier. The step input will be a step in dc voltage to the servo amplifier input. The voltage step should be large enough to cause the motor to reach rated speed. With bldc motors it is not possible to measure the high frequency phase current in a WYE connected motor. Thus some other procedure must be used to measure the current or torque and velocity. Commercial bldc servo amplifiers have two dc output test points. One output is a dc voltage proportional to velocity with a given calibration. The voltage can be directly connected to a strip chart as described previously to measure the motor acceleration, A second test output provides a dc voltage proportional to torque or a percentage of rated torque. This calibrated voltage can also be recorded with a stripchart recorder to observe its maximum value for a step voltage input to the servo amplifier. The inertia can thus be calculated as done previously.

MOTOR INDUCTANCE L

To measure the motor inductance use a low voltage ac source to the motor winding. For a dc motor, apply the ac voltage to the armature winding. For a BLDC motor apply the ac voltage to one pair of the three wires. In both cases measure the voltage and the current. Remember that the BLDC motor is usually connected in WYE. Thus the readings will be line-to-line. You want the phase values for the voltage, so divide the voltage by 2. The impedance of the BLDC motor phase winding is then-

$$Im\ pedance = \frac{phase\ voltage[volts]}{line\ current[amps]} = [ohms]$$

The $impedance = \sqrt{reacc\tan ce^2 + resis\tan ce^2} = [ohms]$

$$\mathrm{Re}\,ac\tan ce[\mathrm{var}\,s] = \sqrt{impedance^2 - resis\tan ce^2}$$

Solve for the reactance from this equation. Note that the phase resistance was measured previously. The inductance can than be calculated from-

$$The\ reac\tan ce\ [\mathrm{var}\,s] = 2\pi\ x\ \ frequency\ x\ induc\tan ce$$

The frequency is probably going to be from the ac source at 60 Hz. Thus-

$$Induc\tan ce[henries] = \frac{reac\tan ce[\mathrm{var}\,s]}{2\pi\ x\ \ frequency[hZ]}$$

METRIC CONVERSIONS

If metric values are used, units conversions are included

The equation to calculate inertia for round objects (such as the motor shaft) is-

SHAFTS,PULLEYS,GEARS

ENGLISH

$J = D[in]^4 \times LGTH[in] \times 7.2 \times 10^{-5} = [lb\text{-}in\text{-}s^2]$

D=Diameter

METRIC

$J = D[cm]^4 \times LGTH[cm] \times 0.077 \times 10^{-6} = [Kg\text{-}M^2]$

The equation to calculate the inertia of round hollow shafts is-

HOLLOW SHAFT-GEAR OR PULLEY

ENGLISH

$J = [D_1^4 - D_2^4] \times LGTH\ [in] \times 7.2 \times 10^{-5} = [lb\text{-}in\text{-}sec^2]$

D = [in]

METRIC

$J = [D_1^4 - D_2^4\ ] \times LGTH\ [cm] \times\ 0.077 \times 10^{-6} = [KG\text{-}M^2]$

D= [cm]

MECHANICAL TIME CONSTANT tm

With the motor constants known, you can then calculate the mechanical time constant-

$$t_m = \frac{\sum \frac{R_{L-L}}{2}}{\frac{K_{e(L-L)}}{1.73} * K_T} \quad [sec] = \quad 0.86 \frac{R_{l-l} * J_{total\ at\ motor}}{K_{e(L-L)} * K_T}$$

Note that we use the phase values instead of line-to-line values. Use half the resistance line-to-line and add some resistance for the wiring from the amplifier to the motor. Usually a factor of 1.35 times the phase value of the resistance is a good approximation.

The equation to calculate the reflected inertia of a lead screw is-

LEAD SCREW

METRIC

$$J_{ref} = \frac{D[cm]^4 \, x L[cm] x 0.00788[Kg/\mathrm{cm}^3]}{16 \; x 6366 \; x N^2} = [Kg - M^2]$$

$$J_{ref} = \frac{D[\mathrm{cm}]^4 \; x \; L[cm]}{N^2} \; x \; 0.077 x 10^{-6} = [Kg - M^2]$$

ENGLISH

$$J_{ref} = \frac{\mathrm{D[in]}^4 \;\; x \; \mathrm{L[in] \; x \; 0.284[lb/in^3]}}{\mathrm{N}^2 \;\; x \, 16 \; x \; 246 \; x \; \mathrm{in}^2} = [lb - in - \sec^2]$$

$$J_{ref} = \frac{\mathrm{D[in]}^4 \, x \, \mathrm{L[in] x \; 7.21 \; x \; 10^{-5}}}{\mathrm{N}^2} = [\mathrm{Lb - in - sec}^2]$$

The equation to calculate the reflected inertia of a rack drive is-

RACK DRIVE

L=lead = in/rev

ENGLISH

$$J_{ref} = \frac{WT[lbs]\, x \sec^2}{386}\; x \left[\frac{L\,[in]}{[rev]} \quad x \frac{[rev]}{[2\pi\, rad]}\right]^2 = \left[\frac{lb\text{-}in\text{-}sec^2}{386\,(2\pi)^2}\right]$$

$$\left[\frac{lb - in - \sec^2}{(2\pi)^2}\, x\, 0.00259\right] =$$

$$J_{ref} = WT[lb]\ x \left[L\left\{\frac{in}{2\pi}\right\}\right]^2 x\ 0.00259 = [lb\text{-}in\text{-}s^2]$$

J_{ref} = WT[lbs] x L^2 x 0.00006569 = [lb-in-sec^2]

METRIC

$$J_{ref} = WT[Kg]\ x \left[L\ \frac{mm}{rev}\, x \frac{rev}{2\pi\, rad}\ x\ \frac{cm}{10mm}\right]^2 = \quad \frac{Kg - cm^2}{(20\pi)^2}$$

$$J_{ref} = Wt[Kg]\ x \left[L \frac{cm}{(20\pi)}\ x\ \frac{M}{(100cm)}\right]^2 = [Kg - M^2]$$

$$J_{ref} = WT[Kg]\ x \left[L\left[\frac{mm}{rev}\right]\right]^2 x\ 0.00000002535$$

$$J_{ref} = WT[Kg]\ x \left[L\left[\frac{mm}{rev}\right]\right]^2 x\ 0.02535\ x\ 10^{-6} = [Kg\text{-}M^2]$$